PB 188 356

STRATIFIED RESERVOIR CURRENTS. PART I. ENTERING STREAMFLOW EFFECTS ON CURRENTS OF A DENSITY STRATIFIED MODEL RESERVOIR. PART II. THE NUMAC METHOD FOR NON-HOMOGENEOUS UNCONFINED MARKER-AND-CELL CALCULATIONS

Larry S. Slotta, et al

Oregon State University
Corvallis, Oregon

October 1969

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Bulletin No. 44 October 1969
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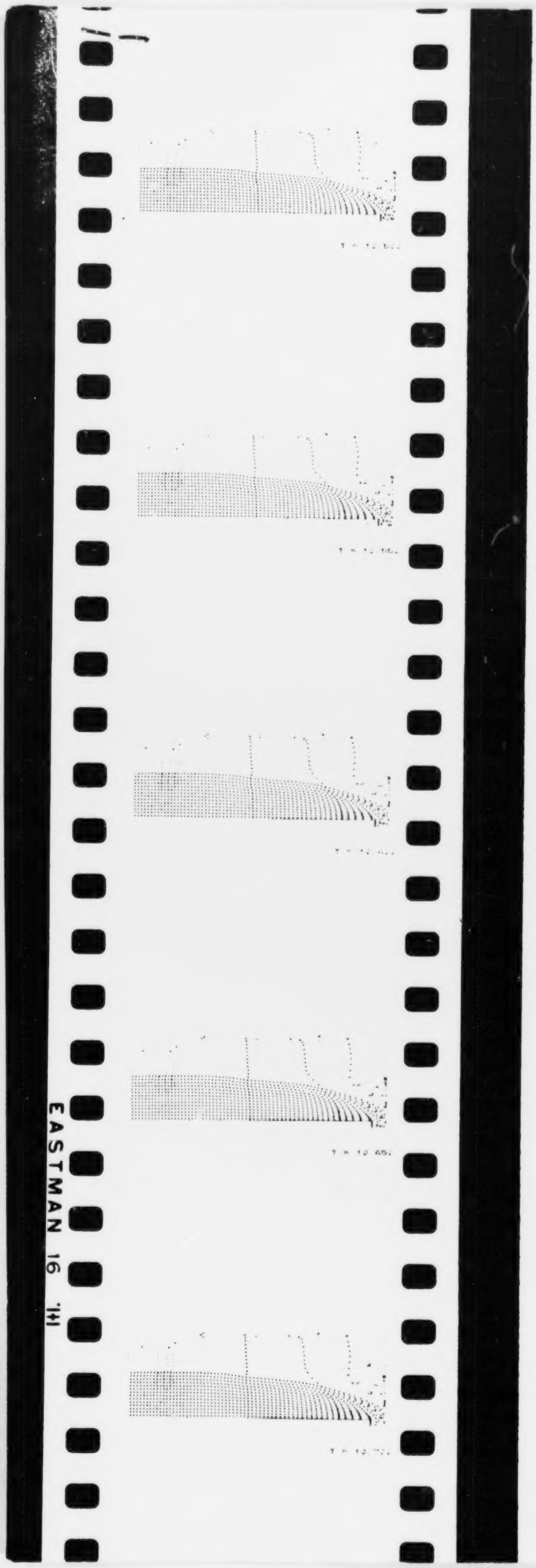

Stratified Reservoir Currents

PART I. Entering Streamflow Effects on Currents of a Density Stratified Model Reservoir

PART II. The Numac Method for Non-homogeneous Unconfined Marker-and-Cell Calculations

Larry S. Slotta
E. Harvey Elwin
Howard T. Mercier
Michael D. Terry

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STRATIFIED RESERVOIR CURRENTS

PART I. ENTERING STREAMFLOW EFFECTS ON CURRENTS OF A DENSITY STRATIFIED MODEL RESERVOIR

PART II. THE NUMAC METHOD FOR NONHOMOGENEOUS UNCONFINED MARKER-AND-CELL CALCULATIONS

by

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BULLETIN NO. 44
OCTOBER 1969

Engineering Experiment Station
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PREFACE

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This volume contains two papers devoted to stratified currents research at Oregon State University under the sponsorship of the United States Department of the Interior, Federal Water Pollution Control Administration. The general purpose of the work was the investigation of the internal currents created by withdrawal from reservoirs stratified by surface heating. Special attention was given to the effects of entering streamflow and withdrawal on currents. The work accomplished during the three years of grant support has been devoted to the following sub-tasks:

1. To examine the effects of topography on the current patterns and waters discharged from a density stratified reservoir.
2. To examine the effects of regulated discharge on stratified current patterns.
3. To consider the influence of entering waters on the current patterns in the pool and subsequent discharge from a stratified reservoir.

Progress has been made in all sub-tasks. Progress reports 1967-1968 have contained graduate degree theses related to the description withdrawal phenomena. Recent presentations have been given:

Hwang, J. D. and L. S. Slotta, 1968, "Numerical Simulation of Selective Withdrawal of Stratified Flows," ASCE Hydraulics Division Conference titled "Computer Applications in Hydraulic and Water Resource Engineering." at M. I. T. Cambridge, Massachusetts, August 1968.

Spurkland, Torbjorn and L. S. Slotta, "Boundary Geometry Effects on Internal Density Currents in a Stratified Reservoir." Pacific Northwest Region American Geophysical Union, Seattle, Washington, October 1968.

Mercier, Howard T., "Digital Simulation in Fluid Mechanics," Pacific Northwest Simulation Council Meeting, Moscow, Idaho, October 1968.

OREGON STATE UNIVERSITY THESES:

Elwin, E. Harvey, 1969, "Entering Streamflow Effects on Currents of a Density Stratified Reservoir." M.S. Thesis, Corvallis, Oregon State University.

Spurkland, Torbjorn, 1968, "The Effect of Boundary Geometry on Internal Density Currents in a Density Stratified Reservoir." M.S. Thesis, Corvallis, Oregon State University.

Terry, Michael D., 1968, "A Numerical Study of Viscous, Incompressible Fluid Flow Problems." M.S. Thesis, Corvallis, Oregon State University.

Mercier, Howard T., 1968, "A Predictor-Corrector Method for the Transient Motion of Non-homogeneous, Incompressible, Viscous Fluid." M.A. Thesis, Corvallis, Oregon State University.

Hwang, J.D., 1968, "On Numerical Solution of the General Navier-Stokes Equations for Two-layered Stratified Flows." Ph.D. Thesis, Corvallis, Oregon State University.

The research outlined as goals of the grant has been advanced on two fronts; one through laboratory studies and the other through numerical or computational approaches. The scope of this work is quite wide, but an effort to stay within limits of the sub-tasks and to significantly contribute to each of the sub-task areas was made. Continued research on the mechanism of stratified currents and selective withdrawal is needed. Research involving field studies in actual reservoirs is necessary to verify predictive behavior as determined in model studies.

Laboratory Studies

Time-lapse photographic techniques for recording flows through a density stratified impoundment model permits viewing a lengthy experiment

(1-1/2 hours) in a few minutes in movie form. Specific studies on geometrical effects of boundaries on internal currents have been conducted. Obstructions such as sea ridges have been placed in the reservoir flow field and the resulting flow patterns observed and recorded. The effect of entering streamflow on currents has also been studied with the laboratory model. Dimensionless parameters have been found which quantitatively relate the existence, location, and magnitude of model internal density currents to the entering streamflow characteristics. Extensions of the model relations for use in the prediction control and maintenance of quality water discharge from actual thermally stratified reservoirs have been proposed.

Field studies in actual reservoirs are necessary to verify the behavior of shear current patterns as predicted from model studies. Additional laboratory investigations should be performed involving surface winds flowing to and counter-current to the reservoir's axis to study possible current reversals. Little research information has been found in the literature that gives attention to wind induced currents on thermally stratified reservoirs. Current reversals caused by surface winds have been generated on a laboratory model. Continued research should be extended to consider the effect of surface wind shear on sub-surface flows. A balance between inlet caused currents and those from counter current winds should give measure to the amount of energy added by each. Field investigations should follow laboratory studies for verifying predictive models.

Analytical Studies

Computer simulation of density stratified flows have been advanced by Oregon State University's approach to density stratified reservoir selective withdrawal problems. Graphic displays of time development of internal stratified flows have been simulated. The computer code NUMAC (Non-homogeneous Unconfined Marker and Cell) is proposed as a valid tool for analyzing transient, incompressible, density stratified or non-homogeneous, viscous flows with a free surface.

Previous analytical and experimental research on the problem of stratified flows has given only limited results which involve either multilayered or continuous density distributions. Nearly all analytical work has undergone simplifications through linearization, boundary-layer approximations, and the use of transitions or geometrical symmetries. The general solution of the complete Navier-Stokes equations governing heterogeneous, time-dependent, incompressible, viscous, laminar flows is sought through numerical methods (Slotta, et al., 1968). Thus by numerical simulation the number of approximations in the mathematical analysis can be minimized, except those arising from the finite difference representations.

The NUMAC method has been applied to simulate selective withdrawal from reservoirs that have: a) two distinct layers of fluids having different densities and viscosities; and b) continuous distribution of density and corresponding viscosity. Results have been found to favorably compare with experimental and analytical data. Other problems which have been simulated with output in movie form include:

One-fluid reservoir with withdrawal.
Two-fluid withdrawal with submerged ridge.
Wave passage over submerged pipe.
Pressure forces on obstacles from wave passage.
Salt water wedge upslope.
Salt water wedge slug flow.
Buoyant pollution plume emitted into a density stratified tank.

An annotated bibliography, "Numerical Methods for Fluid Dynamics", compiled in June 1969 by the Los Alamos Scientific Laboratory Group T-3, points to the significant advance in the past three years in the digital simulation of fluid mechanics problems by listing over 155 references and 48 program codes.

One method devised by Welch et al. (1966) was called the Marker and Cell (MAC) method. In addition to numerically solving the system of partial differential equations which govern the flow of viscous, incompressible fluids, the MAC method demonstrated the use of visual display of the model. Numerical investigators now simulate and watch flows develop as the laboratory investigator might.

The MAC and NUMAC methods use finite difference approximations to the governing partial differential equations. Thus, a differential problem which has no easy analytic solution is approximated by a readily solvable algebraic problem.

The significance of this research is that as better simulation schemes better characterize the flow patterns in water systems, then better water quality management and prediction methods can be generated with these tools. Even though the tools and results presented in this report are significant contributions in the form of simulation technology, extensions of this work are needed. The NUMAC algorithm adequately considers inflows and outflows of density flows through channels; but, some numerical instabilities appear on the free surface during running. It would be advantageous to simulate with the MAC and NUMAC codes at a facility having large memory and high speed capability with unrestricted access so that indeed the researcher could observe displays of developing flows rather than long time turn around on batch process runs.

Acknowledgment

The financial support for research on "Stratified Reservoir Currents" by the Federal Water Pollution Control Administration, U. S. Department of the Interior is gratefully acknowledged. This report is based on studies made under grant WP-00983-03, 16080 DRX.

Extensions to the NUMAC algorithm have been made to illustrate impact pressures of waves on structures. Support for this work was obtained through the National Science Foundation Sea Grant to Oregon State University for the project: Applied Hydrodynamics - Ocean Engineering.

Use of the following computer facilities during the tenure of this study is sincerely appreciated: Oregon State University; National Center for Atmospheric Research, Boulder, Colorado; and Lawrence Radiation Laboratory, Berkeley, California.

My deepest gratitude is directed to the Oregon State University staff who have contributed to this project, especially Mssrs. Hwang, Spurkland, Terry, Mercier and Elwin.

Corvallis, Oregon
September, 1969

Larry S. Slotta

PART I. ENTERING STREAMFLOW EFFECTS ON CURRENTS OF A DENSITY STRATIFIED MODEL RESERVOIR

E. Harvey Elwin
Larry S. Slotta

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ABSTRACT

The effect of entering streamflow on currents of a density stratified reservoir has been studied in a laboratory model to provide insight into the prediction, control, and maintenance of quality water discharge from stratified reservoirs. Experiments were performed using various concentrations of a sodium chloride solution to provide linear density stratifications. Flowfield current patterns and velocities were determined photographically. Flow pattern parameters were found relating the existence, location, and magnitude of model internal density currents to entering streamflow characteristics. The extension of these model reservoir results to prototype conditions is discussed.

Entering Streamflow Effects on Currents of a Density Stratified Model Reservoir

I. INTRODUCTION

In recent years increasing populations with increasing demands of water for municipal and agricultural uses, together with rapidly expanding industrial needs are putting increasing pressure on man's most important natural resource--water. This pressure has been periodically eased by the authorization and construction of an increasing number of impounding reservoirs; however, the total supply of quality water eventually will be limited, and man must learn to use his supplies efficiently.

In order to use a water supply more efficiently, man must be concerned with water quality because the value of a quantity of water is a function of its quality. If man could sort his water supply on the basis of quality, maximum efficiency in reservoir management could be achieved. For example, if man knew how to predict and control the quality and movement of water in a reservoir, the most potable water could be drawn off for domestic needs, the coolest water used for industrial cooling, the warmest water saved for recreation, and the life of impoundments lengthened by using sediment-laden water for irrigation. The quality of conservation flows could be controlled for maximum benefits to fish and wildlife, and short-term polluted flows could be passed through water supplies with a minimum of pollution. Thus, efficient reservoir management is related to the quality and movement of water behind a reservoir.

1. Effect of Impoundment on Quality.

Water quality characteristics may be grouped into three categories: physical characteristics--temperature and turbidity; chemical characteristics--dissolved oxygen, nitrogen, dissolved minerals, and other substances; and biological characteristics--biological oxygen demand, coliform count, and algae count.

Impoundment is among the many things that affect water quality. When a flowing river is dammed and becomes an impoundment, two major changes occur that have a marked effect on water quality. First, an impoundment greatly increases the time required for water to travel the distance from the headwaters to the dam's discharge location. Second, stratification due to density variation in an impoundment changes the characteristics of the water discharged at a given location from what they originally were when the stream was flowing free. Some of the important effects are: a reduction in turbidity; a variation in temperature and dissolved oxygen; and, an increase in algae growth, dissolved solids, nitrogen and phosphorous.

The most important factor in the variation of water quality within a reservoir or lake is a variation in its density. Although density variations or stratification may occasionally be due to chemicals, wastes, or suspended sediments, temperature is analogous in creating density variations. It is well recognized that lakes and reservoirs in the temperate zone undergo a complex seasonal variation in temperature. Typical seasonal and spatial variations of temperature in a deep, temperate climate lake are shown in Figure 1.

During winter and at the beginning of spring,a lake is virtually at a uniform temperature throughout its depth and is essentially homogeneous. During early summer with the coming of warmer weather,a definite temperature profile develops as water near the surface absorbs more energy and is, therefore, warmed faster. Through the summer, heat is absorbed at the surface and mixed downward, largely by wind action with the surface temperature only changing slightly. In late summer a reservoir will have obtained maximum stratification. After this time, as the weather cools, the surface temperature begins to fall creating an unstable condition. Surface water as it cools is more dense than the water beneath it. Overturning occurs and the mixing results eventually in an isothermic condition. The cylical variation of temperature is controlled by various inputs and outputs of energy; solar

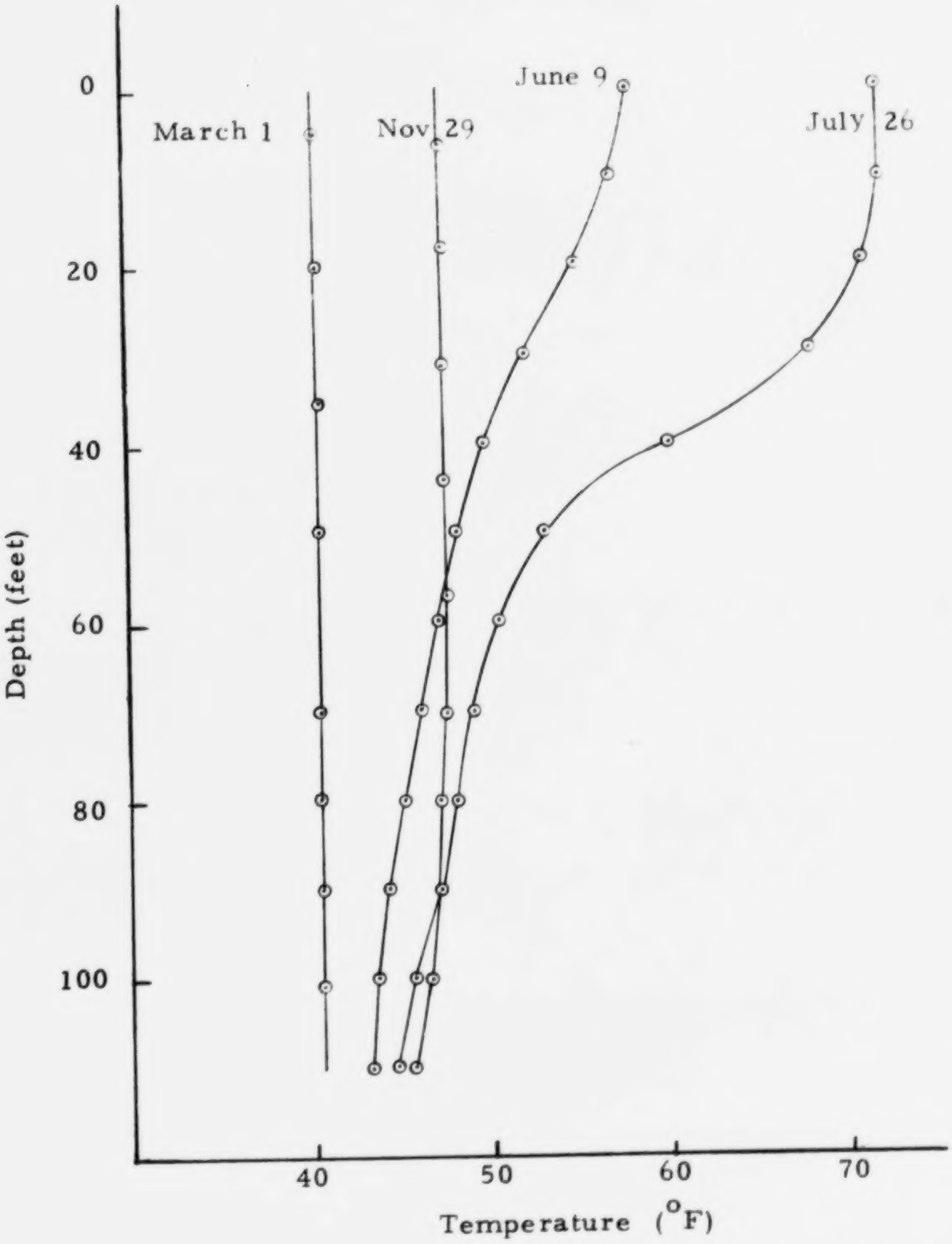

Figure 1. Temperature profiles of the west basin of Horn Lake, B.C., during 1960. (Clay and Fahlman, 1962)

radiation; the convection of heat into and out of the reservoir; evaporation; and back radiation. Analytical and experimental work has been done in an attempt to predict thermal stratification of lakes and reservoirs by Dake and Harleman (9), and an actual method of prediction has been used with good results on Hungry Horse reservoir by Ross and MacDonald (25).

The zone of steep gradient which joins the upper mixed layer (epilimnion) to the cooler body of water below (hypolimnion) is generally referred to as the metalimnion of thermocline. The definitions are illustrated in Figure 2.

Stratification is most important in determining water quality in reservoirs. It may influence water quality through a direct relationship between density and physical or chemical quality parameters, or it may influence water quality by controlling movement of water in the reservoir. The movement of water in the reservoir determines detention time and has an influence on biological quality parameters.

2. Internal Currents

The variations of fluid density in a thermally stratified reservoir give rise to internal flow patterns which may differ entirely from those encountered in homogenous fluids under similar boundary conditions. These flow patterns are known as internal

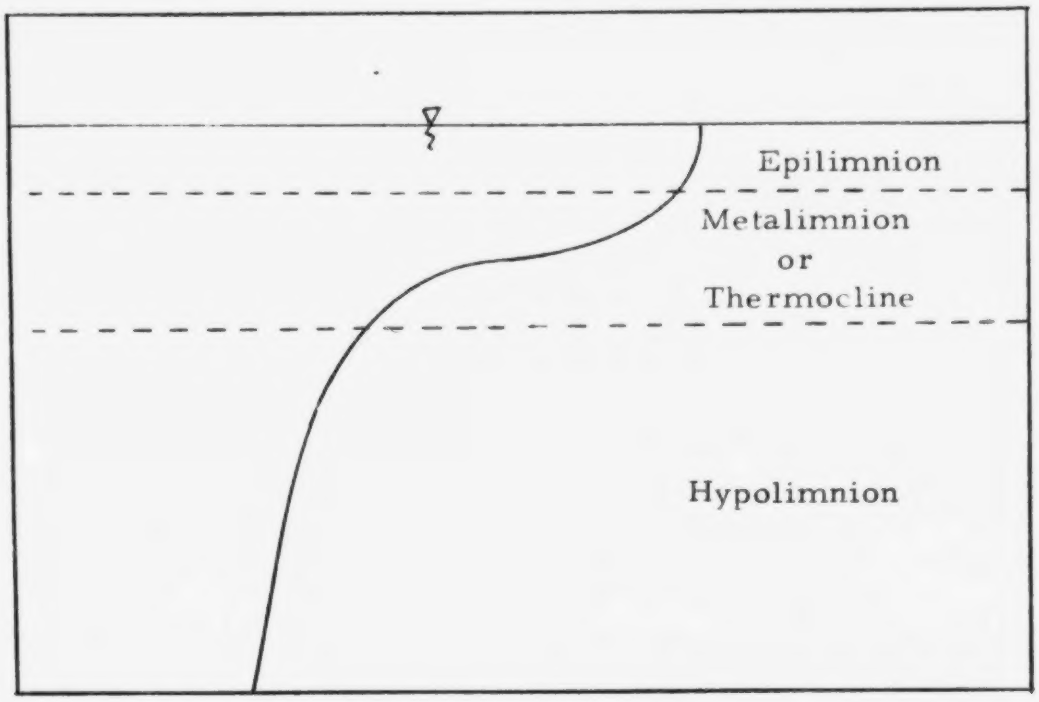

Figure 2. Definition of regions associated with thermal stratification.

density currents.

Internal density currents, although extremely apparent in the flow regime of a reservoir, are not restricted only to reservoirs. A density current may be the gravitationally induced flow of any fluid which is slightly different in density than its surroundings, and the density difference may be due to chemicals, temperature, or suspensions. Interesting cases of density currents may be found in oceanography, hydrology, meteorology, or geology. Ellison and Turner (11) have reviewed some of the situations in nature where nonsuspension density currents occur.

These include the flow of katabatic winds in the atmosphere; the flow of cold water on the ocean floor from arctic to equatorial regions; and the flow of methane fluids along the roof of a mine gallery. Also, Middleton (21) has studied the existence of the turbidity or suspension type density currents over the ocean floor as a means of forming graded offshore beds. Thus this reservoir study has its analog in oceanographic and meteorological investigations.

Density currents in reservoirs are classified by Churchill (6) as three types--overflows, interflows, and underflows. Although Churchill describes these three types of density currents only in terms of the position of the inflowing streams of water, it is recognized that the same types of density currents may be created also by withdrawal from a reservoir. Regardless of whether internal density currents are created by withdrawal or by inflow or by a combination of withdrawal and inflow, they are important to water quality as shown in the following cases.

Density currents exist and cause some unique effects in the Watts Bar reservoir of the TVA system that furnishes the water supply for Harriman, Tennessee (6) . The Harriman water plant intake is located approximately one mile from the upper limit of the backwater on the Emory River arm of the pool and about 13 miles above the junction of the Emory and Clinch arms of the pool. During the winter months, or whenever fairly high flows from the Emory River headwaters exist, the direction of the streamflow for the entire cross section of the pool is downstream from the waterworks. During the summer months, however, when low velocities normally exist, cold water released at Norris Dam into the Clinch River can run upstream in the warmer waters of the Emory arm. As the cold Clinch River water flows up the Emory arm of the pool as a density current, it flows past the Harriman sewer outlets and also past the outfall from a large paper mill. Sewage and mill waste are discharged into the cold water current and are carried by it upstream to the intake of the Harriman water plant, located about one and one-half miles above the paper mill outfall. No one had earlier realized that density currents would extend upstream into the Emory arm of the pool, a distance of 13 miles, but now that they are recognized, the situation has been corrected by using a variable level outfall for the sewage and mill waste.

Turbid density currents have been recognized in America since 1914, when they were reported as having occurred several times in Zuni Reservoir, New Mexico. Most commonly they occur as streamflow entering clear lakes and reservoirs loaded with sediment as a result of floods, but may also result from subsurface landslides. In an early paper, Bell (2) discusses turbidity currents in connection with the sedimentation of Lake Mead. He says the turbidity currents were transporting fine sediments into lower Lake Mead at a rate that will occupy one percent of the original spillway crest capacity each 8.2 years. It is also estimated that by encouraging withdrawal from this turbidity current, much of the sediment may be discharged before it has settled, and that the useful life of Lake Mead could be lengthened by 20 percent in this manner.

In order to increase the production of Pacific salmon, the Canadian Department of Fisheries has established a fish hatchery on the Big Qualicum River in British Columbia. In order to improve conditions for the fishery, it has been considered desirable that a uniform flow of approximately 200 cfs be maintained during the spawning period from late summer to mid-winter. Since the Big Qualicum is at its extreme low flow during the late summer and early fall, a reservoir was established. It was found that under controlled flow conditions, the increased summer minimum flows masked the cooling influence of groundwater sources downstream from the reservoir. In order to keep the stream temperature of the lower river in the ranges optimal for the production of salmon in the July through September period, hypoliminal water is drawn from the lake via low level intake in gradually increasing amounts to temper the epiliminial water drawn from the upper layer. (7)

Thus, the natural temperature regime of the salmon is duplicated, using density currents created by withdrawal.

An organic, bacteriological, or chemical pollutant, if it flows into a reservoir as a density current, may behave as a quasi-pipeline. It has been found that a pollutant discharged from an industrial plant flowed through Cherokee Reservoir of the TVA system as a discrete flow with a minimum of dispersal and diffusion, and the water was discharged through turbine outlets with a minimum of pollution to the reservoir storage.

The previous situations show that the management of reservoir water quality depends in large part on how well one can control the internal current regime in a reservoir.

3. Purpose and Scope of Investigation

Reservoir internal density currents have been studied by theoretical approaches, laboratory experiments, and direct measurements of velocities and stratifications on prototype reservoirs. However, the majority of these efforts have been toward the study of withdrawal currents, and little has been done with inflowing density currents. Since what flows out of a reservoir at one time was streamflow it seems that inlet streamflow effects on reservoir current regimes should merit more consideration.

In the present study, the influences of entering streamflow on the current patterns of a model stratified reservoir are reported. This study is an attempt to relate various parameters of entering streamflow at the upper end of a thermally stratified reservoir to the current regime in the reservoir for the purpose of maintaining quality control.

II. ANALYTICAL CONSIDERATIONS

Presentation of basic assumptions and equations pertaining to two-dimensional, inviscid, steady, incompressible, continuously stratified flow are given in the following section. Withdrawal currents and inflows are next discussed analytically, and finally the method of analysis used to establish the desired streamflow-current regime relationships is explained.

1. Stratified Flow Equations

Consider an incompressible fluid such as water stratified by a slight linear density gradient, as associated with the thermal structure of temperate zone reservoirs or as is created by salinity variation in an estuary. Also consider the flow of any internal currents to be two-dimensional and independent of time where x and y are the respective horizontal and vertical coordinates and u and v the velocity components in the x and y directions.

Figure 3 shows the basic stratified system. With this notation

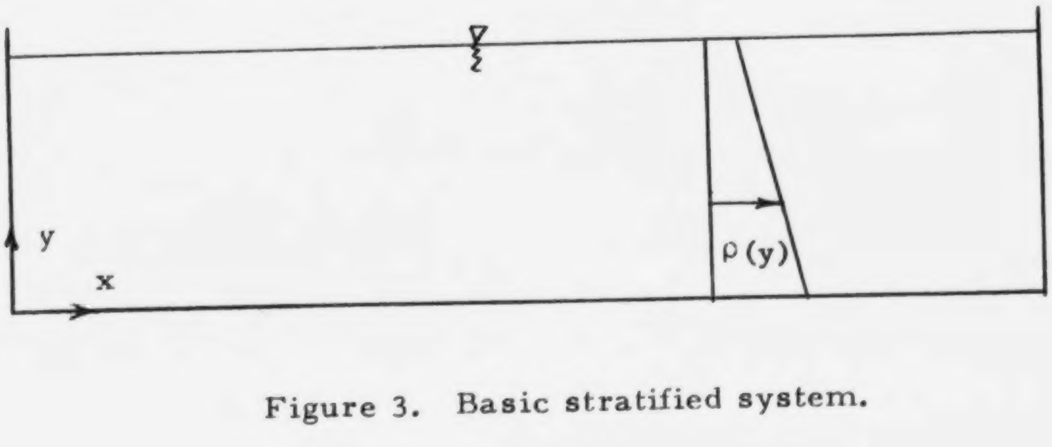

Figure 3. Basic stratified system.

the condition for incompressibility in the sense that a liquid element undergoes a negligible volume change by definition is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad 2\text{-}1$$

The general continuity equation,

$$\nabla \cdot (\rho \overline{V}) + \frac{\partial \rho}{\partial t} = 0,$$

where

$\overline{V} = u\hat{i} + v\hat{j},$

ρ = density,

t = time,

∇ = gradient operator,

is valid for stratification due to temperature variation, but if the stratification is due to a dissolved substance, an additional term is needed to account for mass transfer due to molecular diffusion. Molecular diffusion may be described by an observational law known as Fick's first law in which the rate of mass transfer of a substance per unit area is proportional to the gradient of concentration of the substance. Assuming Fick's first law of diffusion, the mass rate of flux per unit area is:

$$J = -\nabla \cdot [D'\nabla C]$$

where

J = mass rate of flux per unit area,

D' = diffusion coefficient,

C = concentration of substance.

The expanded continuity equation may be rewritten:

$$\frac{\partial\rho}{\partial t} + \rho(\nabla\cdot\overline{V}) + \overline{V}\,(\nabla\cdot\rho) = \nabla\cdot\,[D'\nabla C].$$

From the assumptions of steady, incompressible flow the continuity equation may be simplified:

$$u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} = \nabla\cdot\,[D'\nabla C].$$

Assume a small density variation so that the diffusion coefficient approximates a constant. Also assume a linear relationship between concentration and density so that

$$\rho - \rho_0 = M\,(C - Co).$$

Substituting for C, the equation for the conservation of mass becomes:

$$u\frac{\partial\rho}{\partial x} + v\frac{\partial\rho}{\partial y} = \frac{D'\partial^2\rho}{M\partial y^2}\,. \qquad 2\text{-}2$$

The equations of motion expressing the relationship between the inertial force per unit volume, the pressure force per unit volume, the gravitational force per unit volume, and the viscous force per unit volume are written as:

x-direction: $$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right); \qquad 2\text{-}3$$

y-direction: $$0 = \frac{\partial p}{\partial y} - \rho g; \qquad 2\text{-}4$$

where

p = pressure,

g = gravitational acceleration,

μ = kinematic viscosity.

From the above equations it is apparent that the driving force of internal density currents must stem from the imposition of a pressure gradient into the flow field.

Internal density currents important to a reservoir are associated with the pressure gradient formed by inflowing or outflowing discharges and should be governed by equations 2-1, 2-2, 2-3, and 2-4.

2. Withdrawal Currents

Internal density currents under conditions of withdrawal have been studied extensively in literature, and limited analytical solutions to equations 2-3 and 2-4 under conditions of withdrawal have been attempted through work by Long(19), Yih (31), Kao (16), Koh (17), and Gelhar and Mascolo (15). Long (20) first approached the problem by assuming that the velocities involved were large enough to ignore viscous and diffusive terms. He then simplified the equations of motion to an equation for the stream function. Yih (31) showed that the equation for the stream function could be linearized by defining a transformation. The governing differential equation after transformation by Yih became

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{g\varepsilon\psi}{U^2} = -\frac{g\varepsilon}{\nu}\,y,$$

where

$$\varepsilon = -\frac{1\partial\rho}{\rho_o\partial y},$$

$$\nu = \frac{\mu}{\rho}.$$

Normalizing the equation by the depth d as follows:

$$\xi = \frac{x}{d}\ ; \qquad \eta = \frac{y}{d}\ ; \qquad \theta = \frac{\psi}{Ud}\ ;$$

the equation transforms to:

$$\frac{\partial^2\theta}{\partial\xi^2} + \frac{\partial^2\theta}{\partial\eta^2} + \frac{g\frac{\Delta\rho}{\rho}d}{U^2}(\theta+\eta) = 0,$$

where $\frac{g\frac{\Delta\rho}{\rho}d}{U^2} = Fr^{-2}$, the inverse square of a modified Froude number. Of particular significance was that the critical values for Yih's solution occurred in terms of the modified densimetric Froude number, Fr. Yih found that for $Fr < \pi^{-1}$ this solution no longer upstream boundary conditions. Experiments by Debler (10) qualitatively confirmed the limits of Yih's solution and also demonstrated that where Yih's solution failed the flow patterns were in the form of definite flowing layers separated from nonflowing zones by free streamlines. Kao (16) extended the inviscid solution for $Fr < \pi^{-1}$ by altering the boundary conditions and obtained the equation for the free streamlines along with the velocity distribution. Koh (17) found a solution to the equations of motion, including both viscous and diffusive terms, by perturbation techniques. He analytically described the withdrawal layer and experimentally confirmed his results. Gelhar and Mascolo produced a solution ignoring diffusion by using the same basic assumptions as did Koh.

An example of the solution for the withdrawal layer as done by Koh (17) is shown in Figure 4.

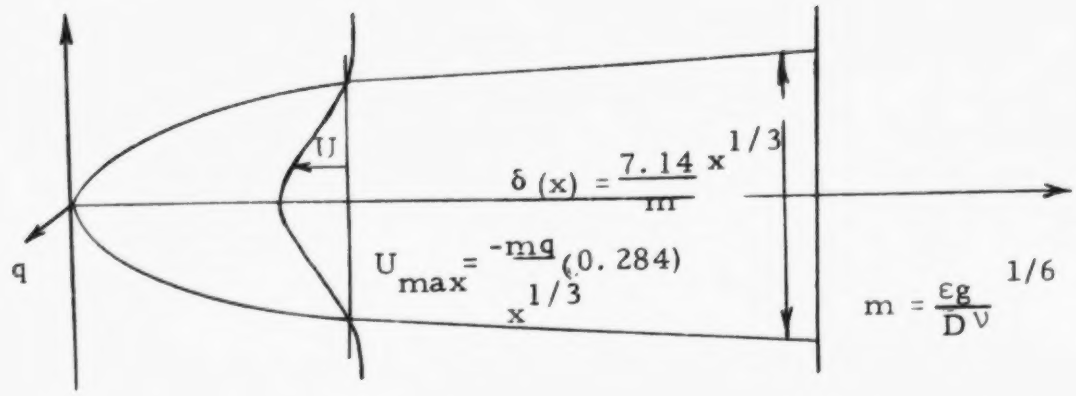

Figure 4. Withdrawal layer toward line sink. (Koh)

3. Inflow Currents

Recent literature concerning discharge into a stratified medium has been concerned with describing inflow parameters and little effort has been made to relate the effect of inflow on the current regime within the stratified medium. However, to analyze the inflow-current regime relationship it is necessary to review basic assumptions concerning the inflow. Literature pertinent to this study concerns the two dimensional turbulent or laminar jet.

Turbulent jet behavior generated by a continuous source of momentum is a fundamental case of free turbulent flows. Development of free turbulent flow in a homogeneous media is discussed extensively in Schlichting (27), Daily and Harleman (8) and Abraham (1). The basic assumptions in most of these treatments consider the conservation of momentum and the assumption of Gaussian velocity and concentration distributions. Extension of free turbulent flow behavior to a stratified ambient fluid has been done by Ellison and Turner (11), Fietz (13), Wada (30), Morton (22), and Fan (12). Ellison and Turner (11) and Fietz (13) studied two-dimensional wall plumes and three-dimensional density currents, respectively, applying largely dimensional analysis techniques. Wada (30) has advanced numerical techniques for the study of cooling water flow patterns from diffusers. Most of the analytical studies of turbulent jets in a stratified fluid have resulted from an integral technique used by Morton, Taylor and Turner (23) in analyzing a simple plume in a linearly density stratified environment. Fan (12) used the Morton type analysis to obtain theoretical solutions for an inclined round buoyant jet in a density-stratified environment.

For this study consider the fully turbulent stream flowing into the density stratified reservoir as shown in Figure 5.

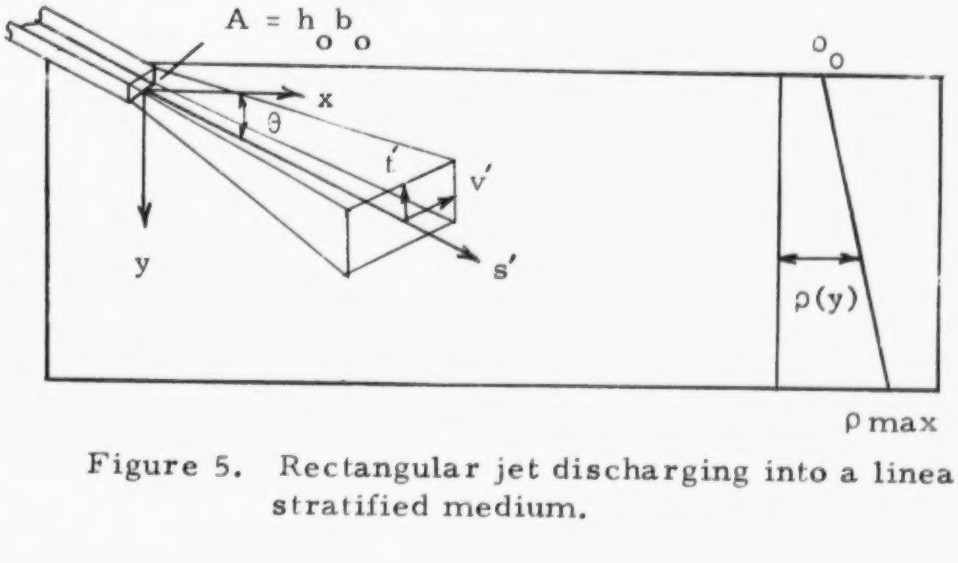

Figure 5. Rectangular jet discharging into a linear stratified medium.

An integral type analysis may be applied. The following assumptions are made:

(i) The fluids are incompressible.

(ii) The velocity distribution is a modified Gaussian distribution modified to a rectangular cross section,

$$u(s', t', v') = u(s') \, e^{\left(-t'^2/h_o^2\right)} \, e^{\left(-v'^2/b_o^2\right)} .$$

(iii) The density of the jet distribution is a modified Gaussian distribution,

$$\rho \, (s', t', v') = \rho \, (s') \, e^{\left(-t'^2/h_o^2\right)} \, e^{\left(-v'^2/b_o^2\right)} .$$

(iv) The rate of entrainment at the edge is proportional to the characteristic velocity,

$$\frac{dQ}{ds'} = (2h+2b) \, ku(s'),$$

where

k = a coefficient of entrainment.

(v) The variation in density is small in comparison with ρ_o.

(vi) Pressure is hydrostatic.

The equation of continuity, based upon the assumed entrainment assumption can be expressed as:

$$\frac{d}{ds} \int_A u(s', t', v') \, dA = \frac{dQ}{ds'} ,$$

$$\frac{d}{ds'}\int_0^{\infty}\int_0^{\infty} u(s')\, e^{\left(-t'/h_o\right)^2}\, e^{\left(-v'/b_o\right)^2}\, dv'dt' = (2h+2b)\, ku(s').$$

Integrating

$$\frac{d}{ds'}\left[u(s')\,\frac{h_o^2}{2}\,\frac{b_o^2}{2}\; e^{\left(-t'/h_o\right)^2}\, e^{\left(-v'/b_o\right)^2}\right]\Bigg|_0^{\infty}\Bigg|_0^{\infty} = (2h+2b)ku(s')$$

$$-\frac{d}{ds'}\left(u(s')\,\frac{h_o^2\, b_o^2}{4}\right) = (2h+2b)\, ku(s'). \qquad 2\text{-}5$$

Since the pressure is assumed to be hydrostatic and there is no other force acting in the horizontal direction, the x momentum flux should be conserved,

$$\frac{d}{ds'}\int_0^{\infty}\int_0^{\infty} \rho(s', t', v')\, u^2(s', t'. v')\cos\theta\, dv'dt' = 0$$

Substituting

$$\frac{d}{ds'}\int_0^{\infty}\int_0^{\infty} \rho(s')\, u^2(s')\, e^{\left(-3t'^2/h_o^2\right)}\, e^{\left(-3v'^2/b_o^2\right)}\cos\theta\, dv'dt' = 0,$$

Integrating

$$\frac{d}{ds'}\left[\rho(s')\, u^2(s')\,\frac{h_o^2\, b_o^2}{9}\; e^{\left(-3t'^2/h_o^2\right)}\, e^{\left(-3v'^2/b_o^2\right)}\cos\theta\right]\Bigg|_0^{\infty}\Bigg|_0^{\infty} = 0.$$

and assuming a small variation in density the following expression is obtained:

$$\text{x-momentum:}\quad \frac{d}{ds'}\left[\frac{\rho o\, u^2(s')\cos\theta\;\; h_o^2\, b_o^2}{9}\right] = 0. \qquad 2\text{-}6$$

In the vertical direction there is a gravity force acting on the jet equal to the change of momentum flux,

$$\frac{d}{ds'}\int_0^\infty\int_0^\infty \rho(s', t', v')\, u^2\,(s', t', v') \quad \sin\theta\, dv'dt'$$

$$= g\int_0^\infty\int_0^\infty\left[\rho(s', t', v') - \rho_a \quad (s', t', v')\right] dv'dt'.$$

Substituting and simplifying

y-momentum: $$\frac{d}{ds'}\left[u^2\,(s')\,\sin\theta\;\frac{h_o^2\, b_o^2}{9}\right]$$

$$= g\;h_o^2\, b_o^2\;\frac{\left[\rho(s') - \rho_a(s')\right]}{\rho_0\,(s')}. \qquad 2\text{-}7$$

From geometry

$$\frac{dx}{ds'} = \cos\theta; \qquad \frac{dy}{ds'} = \sin\theta. \qquad 2\text{-}8 \text{ and } 2\text{-}9$$

The change in amount of dissolved substance in the jet must be conserved with respect to a chosen reference level due to the stability of the density gradient,

$$\frac{d}{ds'}\int_0^\infty\int_0^\infty u(s', t', v')\left[\rho_{in} - \rho\,(s', t', v')\right] \; dv'dt'$$

$$= (2b + 2h)\; k\;\; u(s')\left[\rho_{in} - \rho_a\,(s')\right] \; .$$

Adding and subtracting $\rho_a(s')\, u(s', t', v')$ to the left side and integrating

$$\frac{d}{ds'}\left[(\rho_{in} - \rho_a(s'))\, u \frac{(h_o b_o)^2}{4} + u \frac{(h_o b_o)^2}{9} (\rho_a(s') - \rho(s')) \right]$$

$$= \left[\rho_{in} - \rho_a(s')\right] \frac{d}{ds'}\left(\frac{u\, h_o^2 b_o^2}{4} \right) - \frac{u\, h_o^2 b_o^2}{4} \frac{d\rho a(s')}{ds}$$

$$+ \frac{d}{ds'}\left[\frac{u\, h_o^2 b_o^2}{9} (\rho_a(s') - \rho(s')) \right] .$$

Previously from continuity

$$\frac{d}{ds'}\left[u(s') \frac{h_o^2 b_o^2}{4} \right] = 2\,(h(s') + b(s'))\, k \;\; u(s').$$

Substituting,

$$\left[\rho_{in} - \rho_a(s')\right] 2\, k\, u(s')(h(s') + b(s')) - u \frac{(s')h_o^2 b_o^2}{4} \frac{d\rho a(s')}{ds'}$$

$$+ \frac{d}{ds'}\left[u(s') \frac{h_o^2 b_o^2}{9} \;\; (\rho_a(s') - \rho(s')) \right].$$

$$= \left[\rho_{in} - \rho(s')\right] \;\; 2\, k\, u(s')\, (h(s') + b(s')),$$

the above becomes:

$$\frac{d}{ds'}\left[\frac{u(s') h_o^2 b_o^2}{9} \;\; (\rho_a(s') - \rho(s')) \right] = \frac{u(s') h_o^2 b_o^2}{4} \frac{d\, \rho\, a(s')}{ds'} \quad . \qquad 2\text{-}10$$

With the relationship

$$b_o = m h_o \qquad 2\text{-}11$$

the problem has seven unknowns, namely,

$$u(s'),\ v',\ t',\ \theta,\ x,\ y,\ \text{and}\ \rho_a(s') - \rho(s')$$

and seven equations 2-5, 2-6, 2-7, 2-8, 2-9, 2-10, and 2-11. Initial conditions are:

$$u(o) = U_o;\ t'(o) = h_o;\ v'(o) = b_o;\ \rho(o) = \rho_{in};$$

$$\theta(o) = \theta;\ y = o\ \text{and}\ x = o\ \text{at}\ s = o,$$

but the solution of the system is not obtainable in closed form without the use of numerical techniques and is not presented here.

Very little literature is found (1969) concerning laminar jet flow into a linearly stratified medium, but here too, an approximate analysis may be performed on the inflow by making a few basic assumptions. Consider the case of a density flow proceeding down an incline as shown in Figure 6.

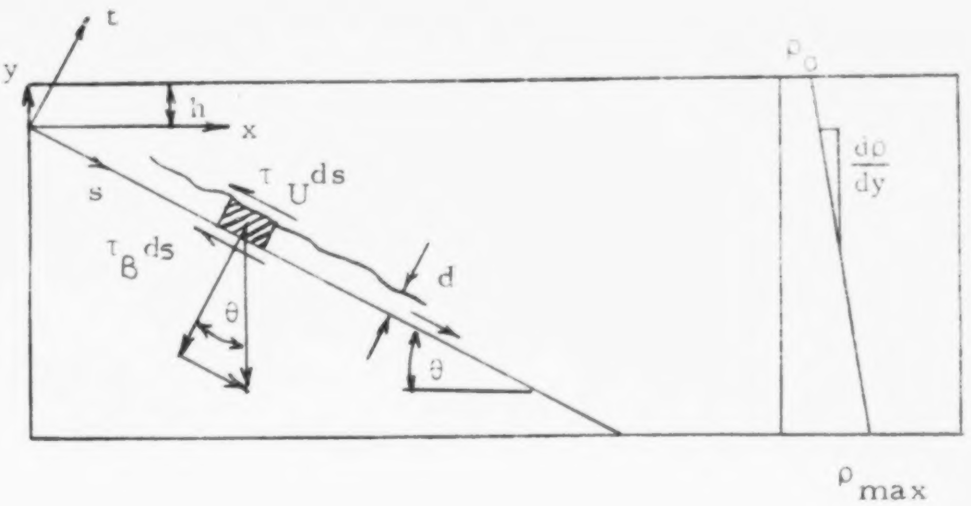

Figure 6. Density flow down an incline.

Assuming in laminar flow that the inertia terms are negligible and that the pressure gradient may be eliminated by cross differentiation, the equation of motion will contain only gravity forces and viscous forces. Summing the forces in the s-direction for the fluid element,

$$W \sin\theta = (\tau_U + \tau_B)\, ds,$$

where

τ_U = surface shear resistance,

τ_B = incline shear resistance,

$W = \left[\gamma in - \gamma_{amb}(s)\right] d\, ds \sin\theta$,

$\gamma_{amb(s)} = g\left[\rho o + (h + s\sin\theta)\frac{d\rho}{dy}\right]$.

and the shear resistance is assumed to approximate the shear relation for pipe flow.

$$\tau = \frac{\gamma in\, f\, V^2(s)}{2g} \quad .$$

Substituting into the force summation,

$$g\left[\rho in - (\rho o + (h + s\sin\theta)\frac{d\rho}{dy})\right] d \sin\theta\, ds$$

$$= \rho in \frac{(f_U + f_B) V^2(s)\, ds}{2},$$

$$V(s) = \left[\frac{2g\, d\sin\theta}{\rho in\, (f_U + f_B)}\left[\rho in - (\rho o + (h + s\sin\theta)\frac{d\rho}{dy})\right]\right]^{1/2} .$$

a relationship is obtained for $V(s)$. Its use, however, is questioned due to the difficulty of evaluating friction coefficients, f_U and f_B. The point at which the density flow leaves the slope is obtained by the criteria that $V(s) = 0$,

$$V(s) = 0 \quad \text{when} \quad \rho in - \left(\rho o + (h + s\sin\theta) \frac{d\rho}{dy} \right) = 0,$$

or referenced from the water surface elevation where

$$\text{depth} = h + s\sin\theta \ ,$$

$$h_{curr} = (\rho in - \rho o) \frac{dy}{d\rho} .$$

This expression shows that the inflow will seek an elevation corresponding to its own density, and agrees with results that Spurkland (28) obtained with a submerged diffuser.

4. Present Study

It was reported in Section 2 that from the governing equations an analytical description of internal density currents due to the imposition of a simple pressure variation may be made. In Section 3 it was shown that in some cases an inflowing jet may be discussed analytically if the appropriate assumptions are made. However, complete solutions are untenable when the relationship between both the inflow and the internal current regime are desired. The interaction among density, velocity, and pressure fields of the inflow and ambient fluid cause the general solution to become very mathematically complex. For this reason the density stratified reservoir flow phenomena are to be analyzed experimentally using a dimensional analysis to find correlation among the physical variables involved in this study.

Consider a streamflow entering a stratified medium with an equivalent outflow rate to maintain a constant water surface level as illustrated in Figure 7a. The independent parameters involved are those describing

(i) Boundary conditions:

D = total depth of reservoir

ϕ = angle of inflow

θ = angle of reservoir slope

h_{in} = depth of slope change

h_{out} = depth of outlet

L = length of reservoir

(ii) Inflow:

Q_{in} = inflow rate

V_{in} = inflow velocity

ρ_{in} = inflow density

b_{in} = inflow width

d_{in} = inflow depth

(iii) Outflow:

Q_o = outflow rate

V_o = outflow current velocity

ρ_{out} = outflow density

d_o = outflow diameter

(iv) Ambient fluid:

$\frac{\Delta\rho}{\Delta y}$ = density gradient

ρ_o = surface density

ρ_{max} = bottom density

(v) Miscellaneous:

g = gravitational acceleration

ν = kinematic viscosity

t = time

The dependent factors involved are those parameters describing the resulting current regime. They are:

$h_1, h_2, h_3, \ldots$ the heights of various currents

$v_1, v_2, v_3, \ldots$ the velocities of various currents

The densities of various currents are not included because they are related directly to the current heights.

It is known that a particular density current will be a function of the independent variables involved:

$$V_{curr} = f\left(D, S_v, S_r, h_{in}, h_{out}, L, Q_{in}, Q_o, \rho_{in}, d_{in}, \rho_o, d_o, \frac{\Delta\rho}{\Delta y}, \rho_{max}, \nu\right)$$

$$h_{curr} = f\left(D, S_v, S_r, h_{in}, h_{out}, L, Q_{in}, Q_o, \rho_{in}, d_{in}, \rho_o, d_o, \frac{\Delta\rho}{\Delta y}, \rho_{max}, \nu\right)$$

and the complexity of establishing a particular relationship is apparent from the number of parameters involved. In order to simplify the analysis, a number of the independent variables as shown in Figure 7b will be held constant. Once the flow configuration becomes known, the number of parameters involved will be further reduced in number by individually considering each main internal current allowing nonpertinent parameters to be disregarded. The functional relationships will be established in chapter IV.

(v) Miscellaneous:

g = gravitational acceleration

ν = kinematic viscosity

t = time

The dependent factors involved are those parameters describing the resulting current regime. They are:

$h_1, h_2, h_3, \ldots$ the heights of various currents

$v_1, v_2, v_3, \ldots$ the velocities of various currents

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$$h_{curr} = f\left(D, S_v, S_r, h_{in}, h_{out}, L, Q_{in}, Q_o, \rho_{in}, d_{in}, \rho_o, d_o, \frac{\Delta\rho}{\Delta y}, \rho_{max}, \nu\right)$$

and the complexity of establishing a particular relationship is apparent from the number of parameters involved. In order to simplify the analysis, a number of the independent variables as shown in Figure 7b will be held constant. Once the flow configuration becomes known, the number of parameters involved will be further reduced in number by individually considering each main internal current allowing nonpertinent parameters to be disregarded. The functional relationships will be established in chapter IV.

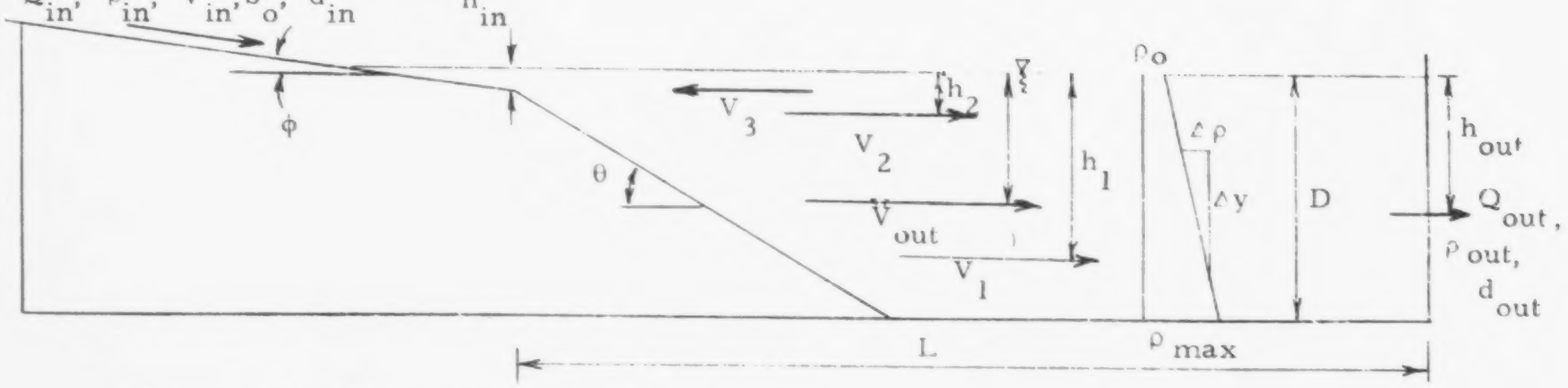

Q_{in}, ρ_{in}, V_{in}, b_o, d_{in}
h_{in}
ϕ
θ
V_3
V_2
V_{out}
V_1
h_2
h_1
ρ_o
$\Delta\rho$
Δy
D
h_{out}
Q_{out},
ρ_{out},
d_{out}
L
ρ_{max}

Figure 7a. Parameters involved in the investigation.

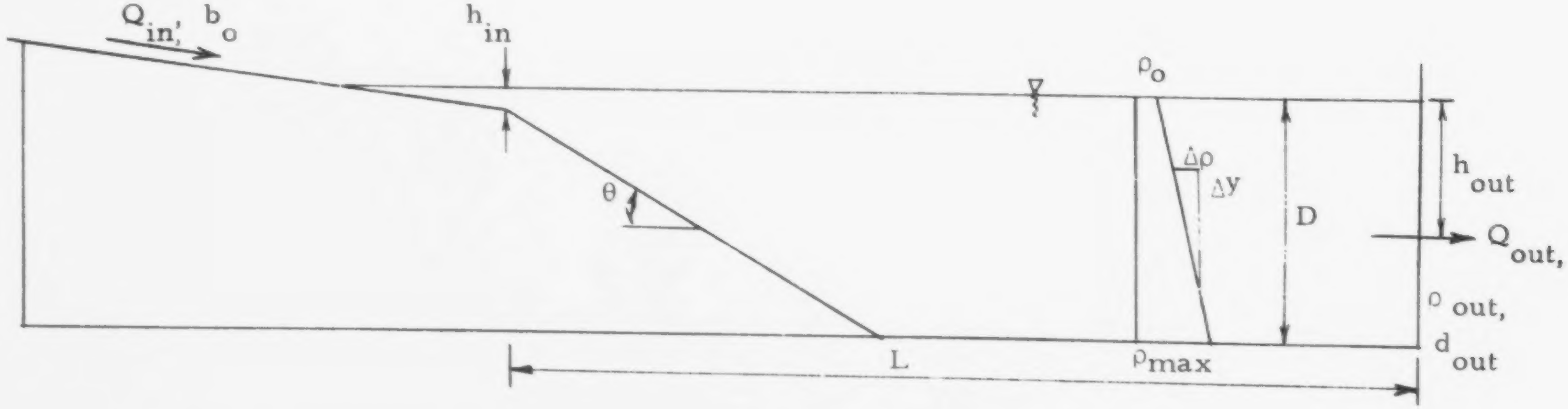

Q_{in}, b_o
h_{in}
θ
ρ_o
$\Delta\rho$
Δy
D
h_{out}
Q_{out},
ρ_{out},
d_{out}
L
ρ_{max}

Figure 7b. Independent parameters held constant during the investigation.

III. APPARATUS AND PROCEDURE

To investigate the influence of entering streamflow on the current regime of a model density stratified reservoir a series of laboratory experiments was performed in which fluid was allowed to enter a tank of stratified fluid by way of a model streambed.

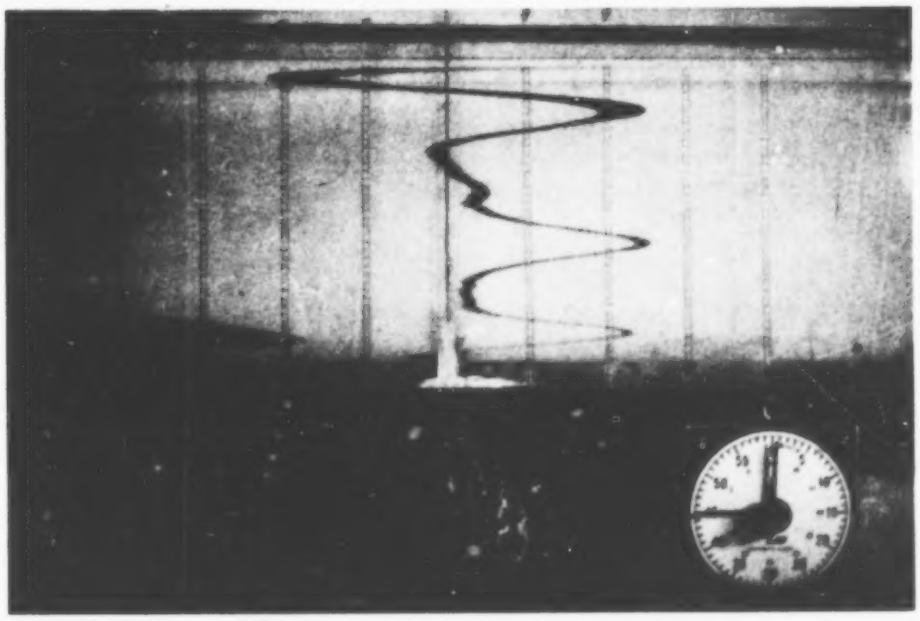
In this chapter the experimental procedure and apparatus used for the experiments will be discussed. The individual steps in the experimental procedure will be explained in detail.

1. General Description of the Procedure

For the series of experimental runs, the model reservoir was first filled with distinct layers of water containing appropriate quantities of salt (NaCl) in suspension to give a linear density gradient from the top to bottom levels of the tank. The water was then allowed to stand several hours so that the density profile would become linearly smooth by molecular diffusion. The density profile was measured indirectly shortly before each run, and after each run by measuring the electrical conductivity of the solution at various levels in the reservoir. Salt solution was mixed with water in the inflow storage tank until the desired inflow conductivity was reached. Inflow and discharge rotameters were opened and the flow rates adjusted to be equal. After waiting for the system to reach a steady state (five minutes), dye (Erioglaucine A Supra) was injected into the inflow fluid in order to trace its movements through the model. To observe the current patterns within the model, dye particles were intermittently dropped into the model reservoir at a reference station. As the dye particles fell, they left a distinct vertical time line. Thirty-five millimeter slides taken at various time intervals and a time lapse movie camera recorded the horizontal motions of the time lines. Typical exposures are shown in Figure 8. An overhead movie camera photographed at various time intervals the entering inflow configuration and its travel. Each run lasted two hours at which time the tank was drained, washed, and set up for the next run. The necessary velocity and configuration measurements were obtained from the film record.

2. The Model Reservoir and Model Stream.

The reservoir for the inflow experiments was a clear walled, rectangular, plexiglas flume. It was 25 feet long, 18 inches wide, and 22 inches deep. A schematic drawing and a photograph of the reservoir are shown in Figures 9 and 10, respectively. The inlet end was equipped with an adjustable bottom slope so that the depth varied from zero to full depth at different possible choices of slope.



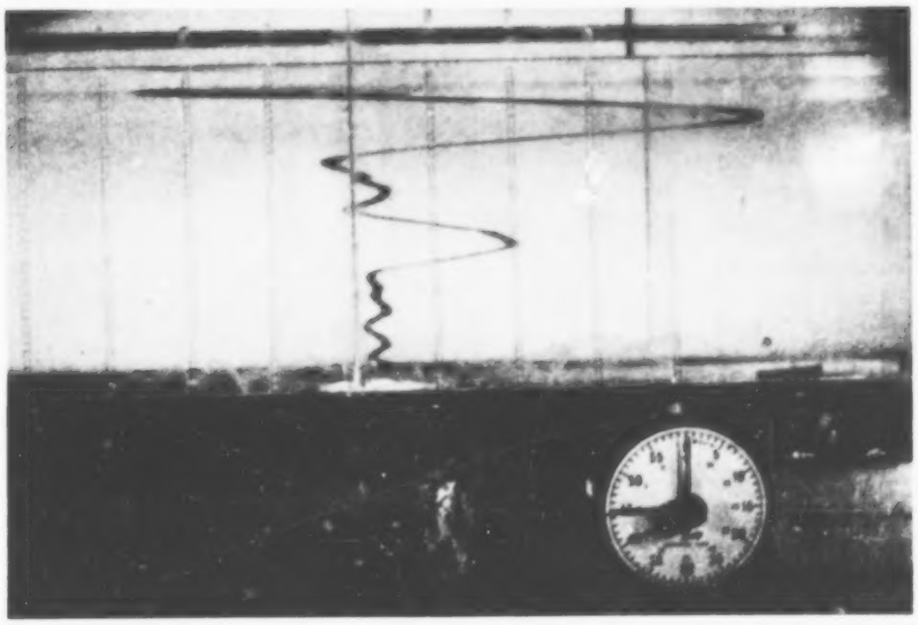


Figure 8. Typical photographs of time lines.

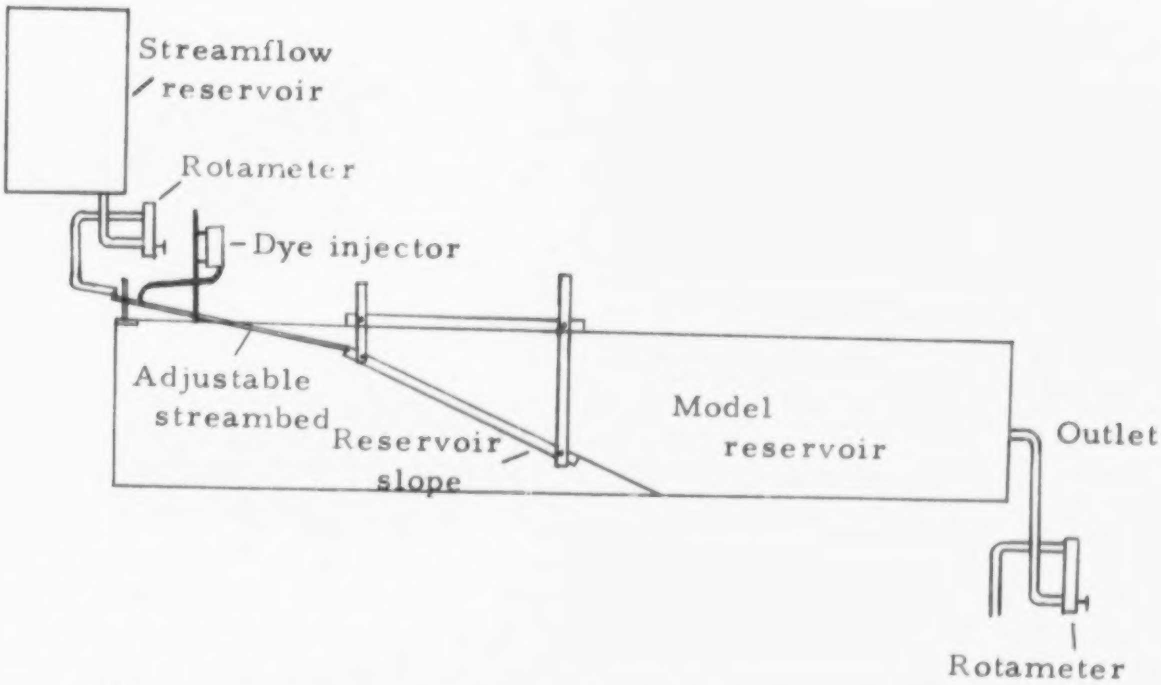

Streamflow reservoir
Rotameter
Dye injector
Adjustable streambed
Reservoir slope
Model reservoir
Outlet
Rotameter

Figure 9. Schematic plan of model reservoir and streambed.

The simulated stream channel was a six foot length of 2"x1"x1/8" aluminum channel mounted on a sheet of plexigas which fit snugly in the width of the tank. The aluminum channel and plexiglas sheet was used as a second slope extending from the end of the tank to the top of the bottom slope. The configuration of two slopes was necessary to provide a continuous slope from above the water surface to the bottom of the tank while maintaining a flat slope for the simulated streambed. The flow for the simulated stream was provided by a storage tank at the upper end of the model reservoir. The water from this tank was released at the upper end of the model stream. The stream was lined with cemented sand grains to provide artificial roughness.

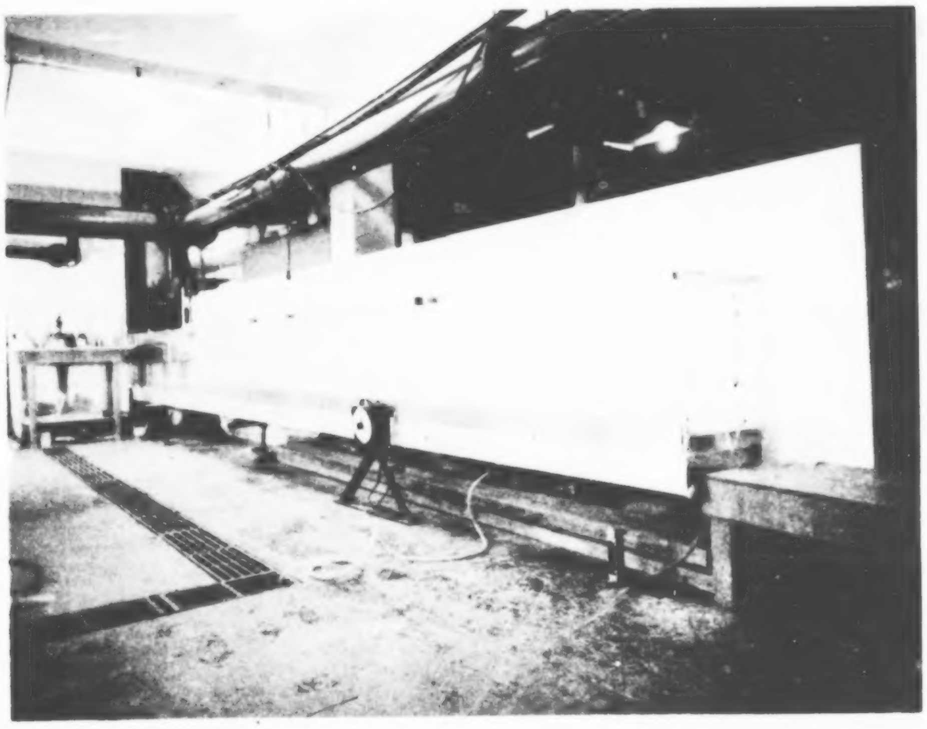


Figure 10. Photograph of model reservoir and steambed.

3. The Filling Apparatus and Procedure

The desired linear density profile was achieved by mixing measured amounts of a saturated salt solution with a fixed amount of water in a mixing tank and placing the mixture in the reservoir. The basic apparatus by Spurkland (28) was redesigned and used for this purpose.

A typical filling cycle began with the activation of a timing cam system by a Lapine multispan timer which was set to provide power for the duration of the filling cycle. Each mixing cycle lasted 40 minutes and involved the opening and closing of the salt tank, water supply, and mixing tank solenoids. The amount of salt brine for each ten mixing cycles was controlled by ten 20-minute sequential timing cams, each activated by a 40 minute cycle timing cam and a pressure switch that shut the water off when the water surface reached a certain level. The draining of the mixing tank was accomplished by another 40 minute cycle timing cam calibrated to the draining time of the mixing tank. A block diagram of the automatic filling apparatus is shown in Figure 11. The salt solutions were introduced into the model reservoir by gravity flow through three stand pipes placed on the floor of the tank. The model reservoir was set on a very mild slope. As additional inflowing layers are

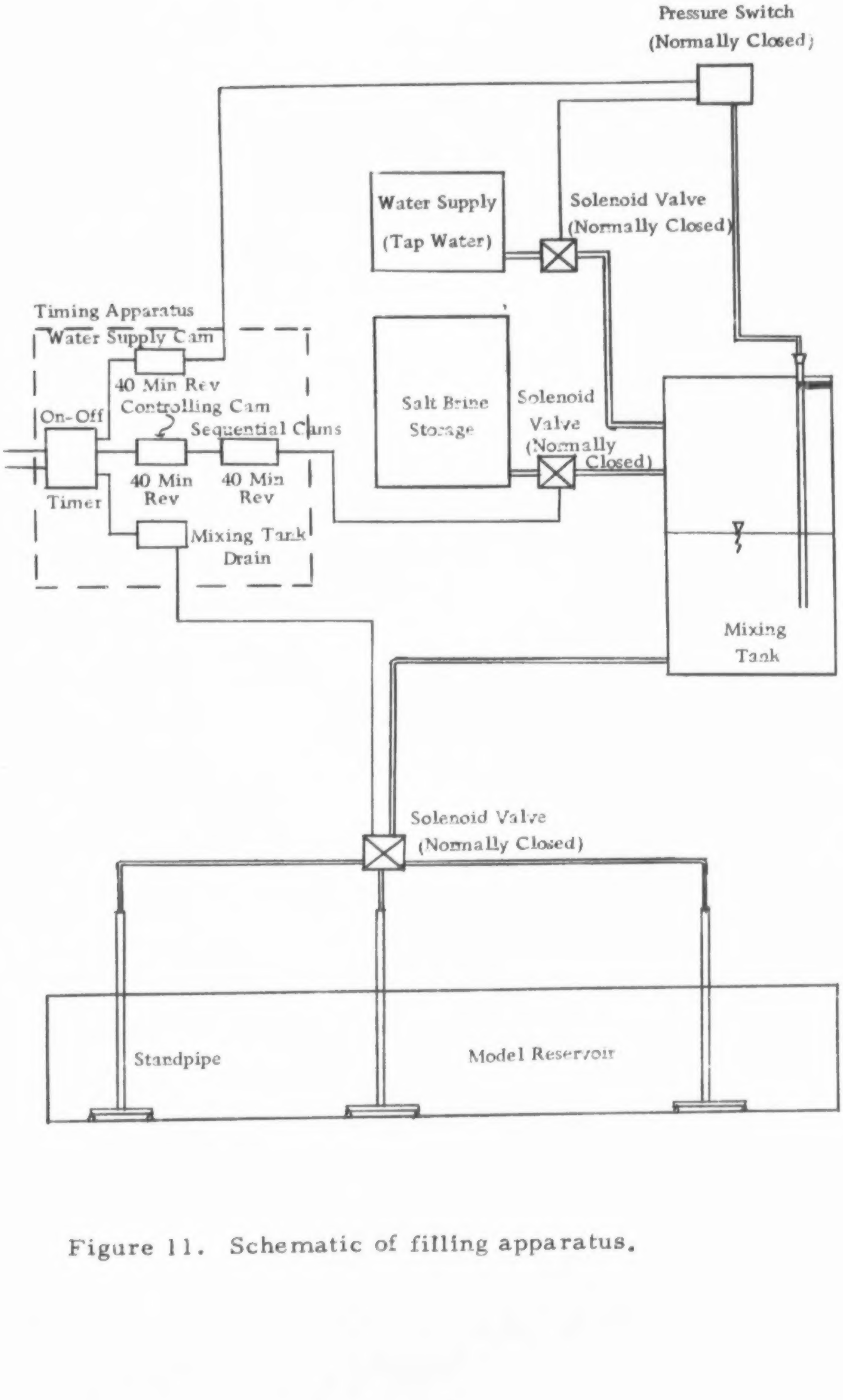

Pressure Switch
(Normally Closed)
Water Supply
(Tap Water)
Solenoid Valve
(Normally Closed)
Timing Apparatus
Water Supply Cam
40 Min Rev
Controlling Cam
On-Off
Sequential Cams
Salt Brine Storage
Solenoid Valve
(Normally Closed)
40 Min Rev
40 Min Rev
Timer
Mixing Tank Drain
Mixing Tank
Solenoid Valve
(Normally Closed)
Standpipe
Model Reservoir

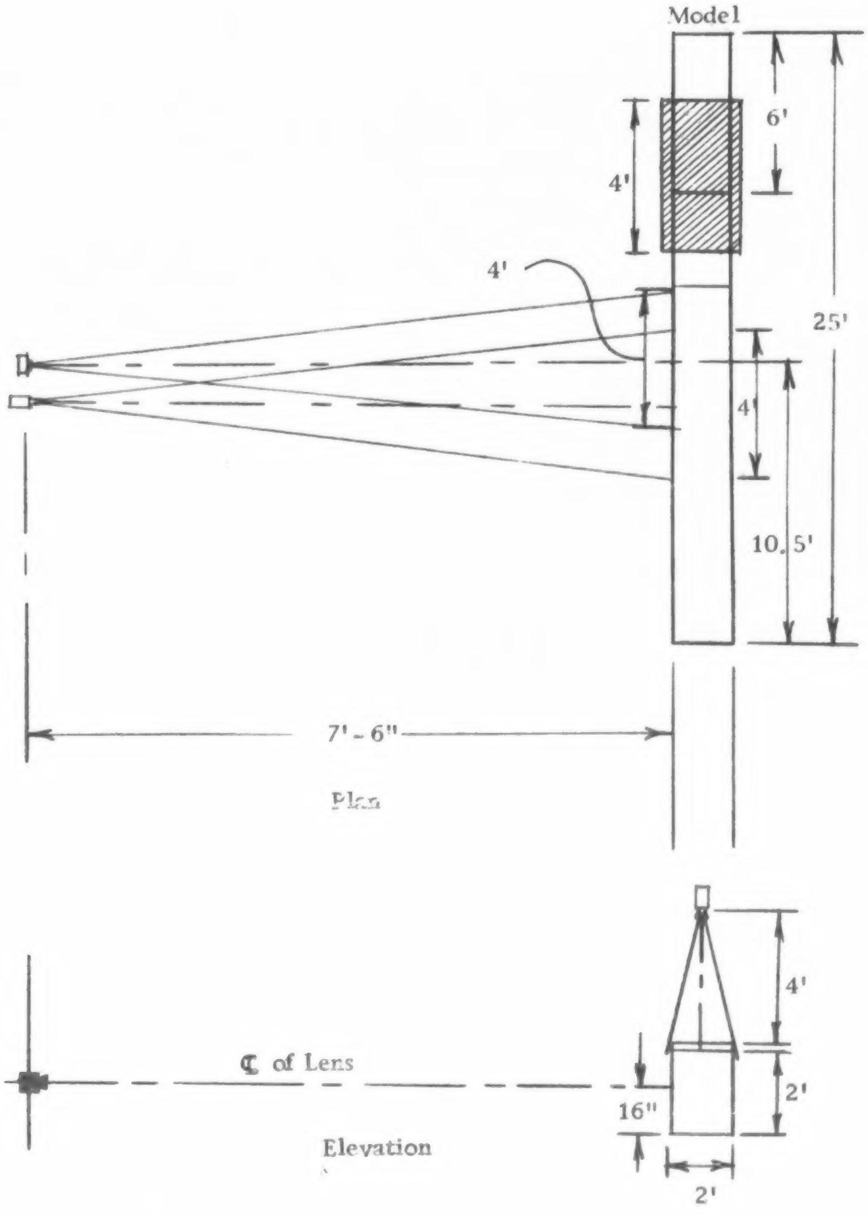
Figure 11. Schematic of filling apparatus.

progressively more dense, they flow slowly by gravity along the bottom of the channel under the other layers creating a minimum amount of mixing.

4. Photography

Because of the complexity of the events during each two hour run, photography was used to record much of the data. An Argus C-3 35mm camera and a Nizo S-80 super 8mm camera were used to photograph the vertical dye streaks, and another Nizo S-80 super 8 mm camera was mounted overhead to observe the inflow configuration. All cameras were used with Kodachrome II color film at ASA 40 in conjunction with photoflood lights. The 35 mm camera had a 50mm Argus Cintar f3. 5 lens while the 8mm cameras had a 10mm-80mm zoom f2. 8 lens which was used at 10 mm.

The tank had a 12:1 length to depth ratio, so the cameras field of view covered a limited area. A reference station was established 10. 5 feet from the mouth of the model stream, and the horizontal cameras were positioned in respect to it. A clock mounted near the wall of the model reservoir gave elapsed time as recorded on film. An overhead camera was positioned over the model stream mouth. A schematic drawing of the positioning and coverage is shown in Figure 12.


Model
6'
4'
4'
25'
4'
10.5'
7'-6"
Plan
℄ of Lens
4'
2'
16"
Elevation
2'

Not Drawn to Scale

Figure 12. Positioning of cameras and respective fields of view

5. Measurement of Density Profiles

A conductivity probe and a Serfass Conductivity Bridge was used to measure the electrical conductivity of the salt solution as a measure of its density prior to and after every run. Several investigators have used the exposed conductivity probe in conjunction with a conductivity bridge with much success as seen from Spurkland (28) Lofquist (18) , and Rumer (26) . From their conclusions it is desirable to use a small platinized probe so that polarization and capacitance effects would be minimized. The probe used in this study was made of two $1 cm^2$ platinum plates, spaced one cm apart as shown schematically in Figure 13. The probe was connected to the conductivity bridge by leads running through a water-tight glass tube indexed in a

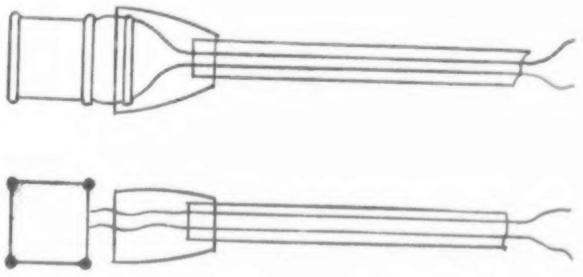


Figure 13. Conductivity probe.

centimeter scale. The conductivity was measured vertically at two-centimeter intervals in the centerline of the tank. To obtain conductivity-density relationship the probe was periodically calibrated with a Christian Becker balance,reading specific gravity directly. A typical density profile and the corresponding calibration curve are shown in Figure 14.

6. Measurement of Flow Rates

After the conductivity profile had been measured, a Brooks rotameter was adjusted at both the inflow and discharge ends of the model reservoir to maintain a constant inflow and outflow rate of 12.6 cubic centimeters per second. Since the rotameters were originally calibrated for a specific gravity of 1.000, they were re-calibrated for each of the five specific gravity values used in this study. The calibration is shown in Figure 15. Although this plot indicates a small density influence on the flow rate, it is small enough relative to the error inherent in reading the rotameter that it may be ignored.

7. Measurement of Velocities

After the flow attained a quasi-steady state (five minutes), potassium permanganate crystals mixed with carbon tetrachloride were dropped into the model reservoir at the reference station

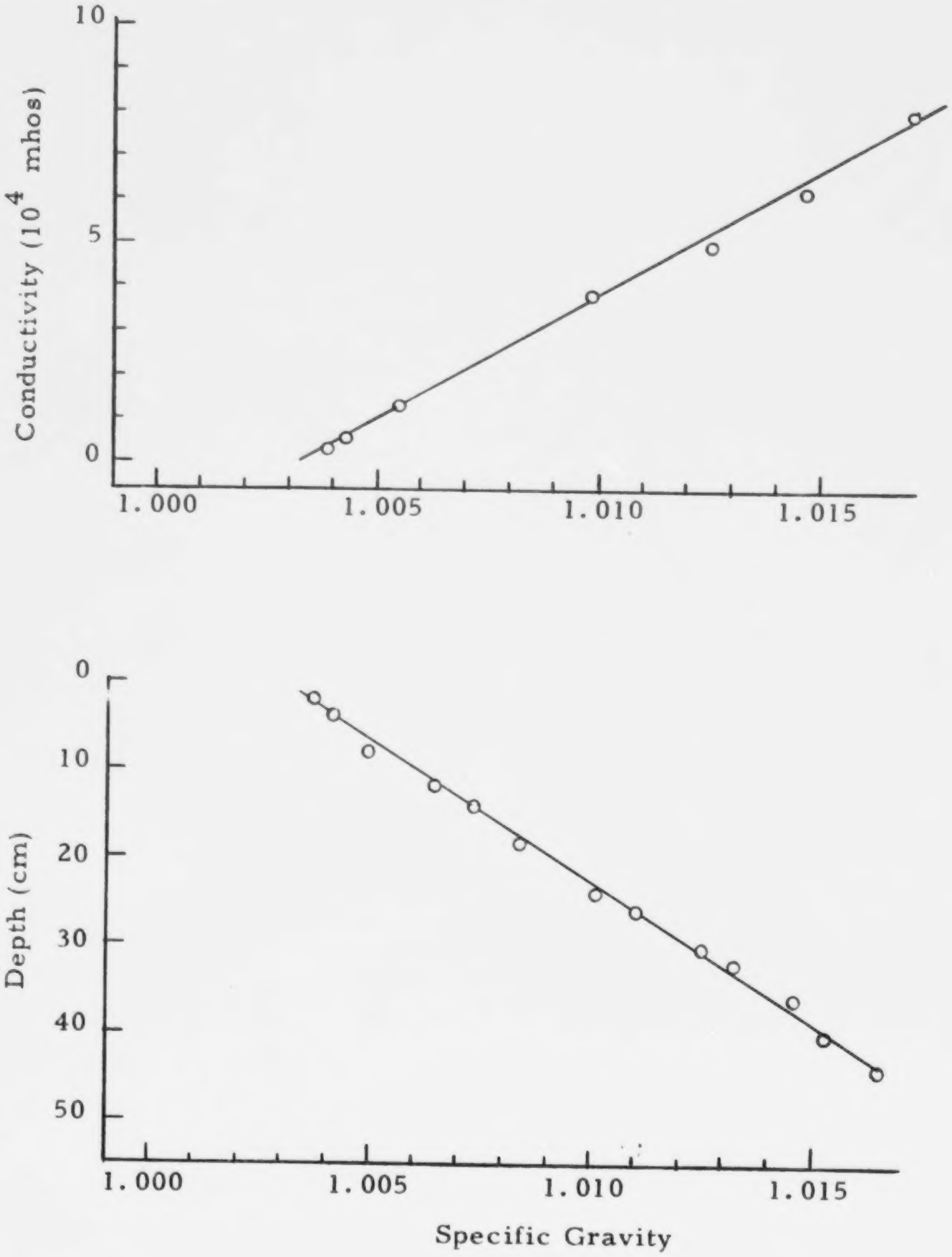


Figure 14. Calibration curve and density profile for run number 21.

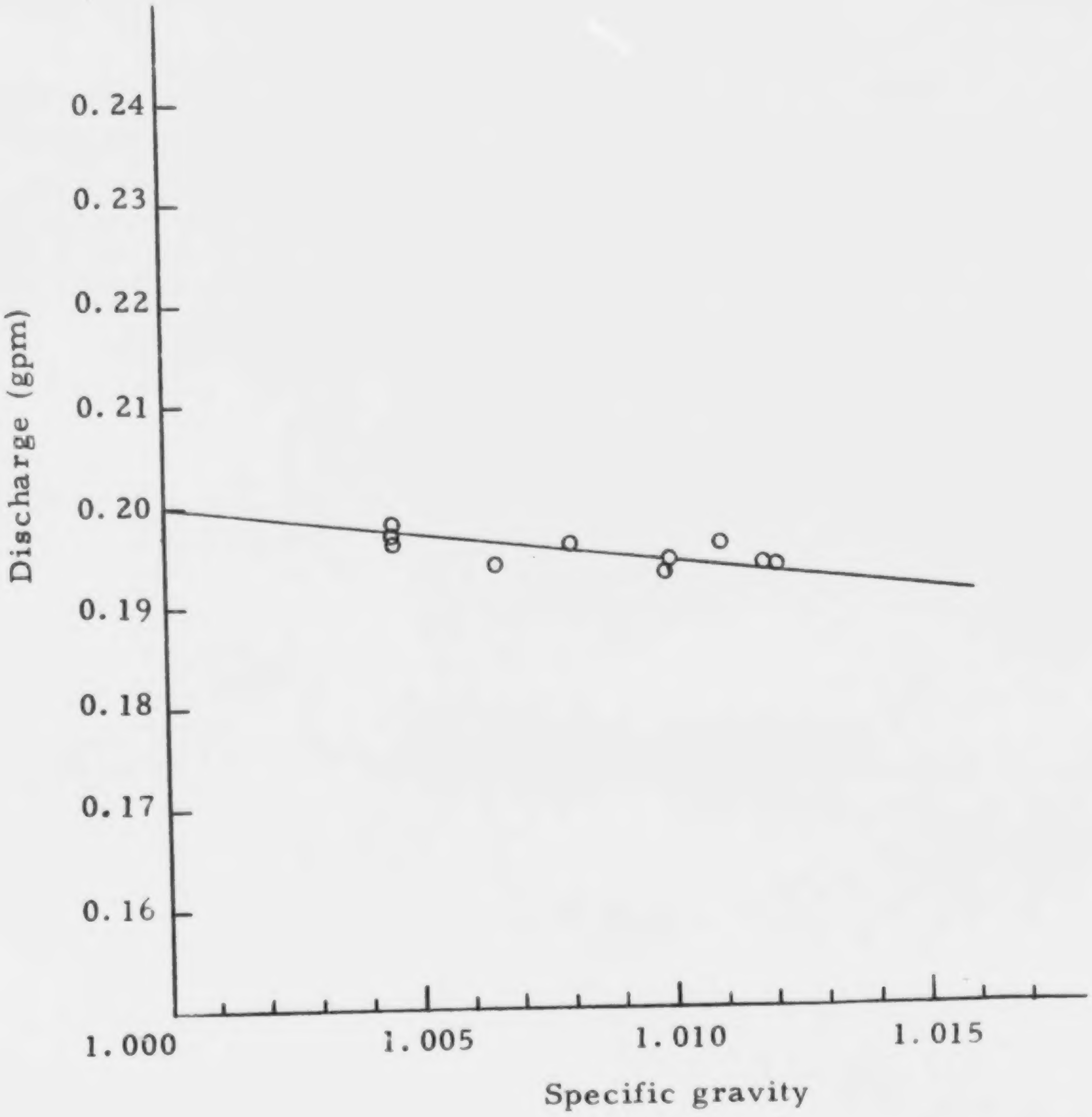


Figure 15. Calibration of inflow rotameter.

forming time lines which deform with the currents. A new time line is injected every 20 minutes for the two-hour period. At least 20 slides were taken at regular time intervals and the movie camera was run continually at one frame every two seconds. After the film was developed, the frames were projected into a viewing box constructed as shown in Figure 16. Time of travel measurements were taken from a grid after establishing the scale of the image projecting the picture distance between the flume's bolts at a constant scale. Measurements were taken near the center of the projected area to minimize parallax.

The overhead camera was operated at 18 frames per second during four intervals in the two-hour run. Time of travel measurements of the inflow stream velocity and the inflow density current were obtained by projection and frame counts.

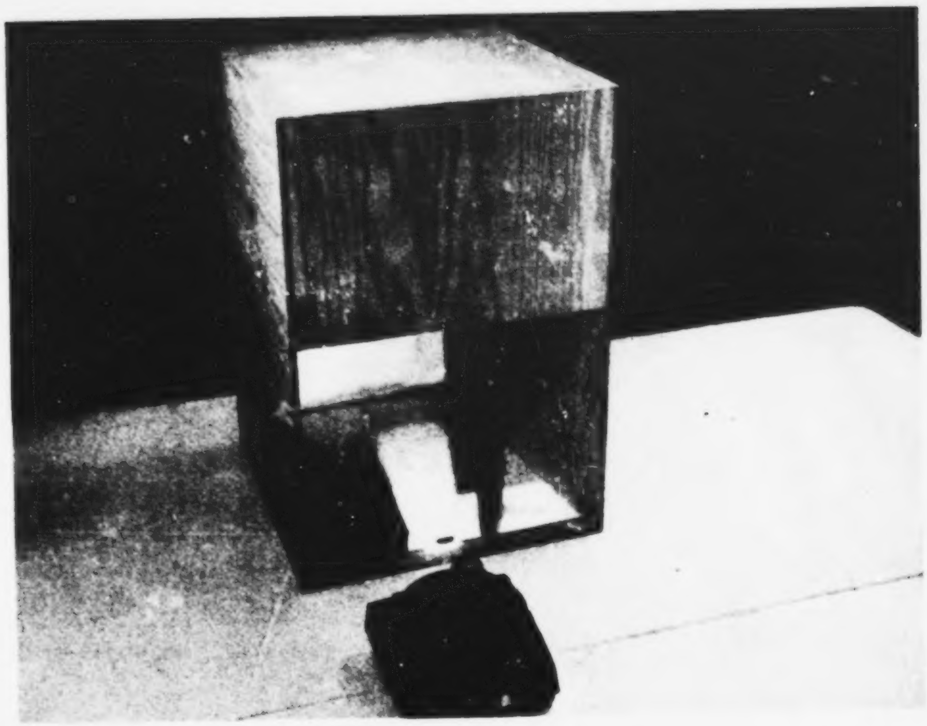


Figure 16. Projection apparatus for viewing time lines.

IV. EXPERIMENTAL RESULTS

Experimental runs were performed with the previously described apparatus to determine the relationship between the entering streamflow and the model reservoir current patterns. The resulting current regime produced in the model reservoir is described, and correlations are established between the observed current parameters and the inflow characteristics for each of the main currents.

1. General Current Patterns

Major repetitive current patterns were created as the entering streamflow, designed Q_{in}, flowed down the sloping streambed and entered the initially static, density-stratified, model reseroir. At the lowest streamflow velocities, V_{in}, little mixing occurred between the ambient fluid and the streamflow, and the majority of the streamflow density current proceeded down the reservoir slope until reaching a reservoir depth having equivalent density. At this point the streamflow density current flowed horizontally across the reservoir and became the main inflow current, Q_1. At the higher streamflow velocities more mixing occurred creating a large mixing current, Q_3; and at the highest streamflow velocities, mixing was so extensive that very little of the entering streamflow discharged down the reservoir slope. As the mixing current, Q_3, increased, a reverse current at the surface, Q_4, caused by entrainment to the mixing current occurred, and an eddy in the vicinity of the stream mouth was consistently formed. A fourth current, Q_2, was formed by the outflow necessary to keep the water surface elevation constant. A typical or general current pattern existing in the model reservoir during a test run is indicated in Figure 17.

Occasionally small intermediate currents were noticeable between the major currents shown in Figure 17, but these were relatively minor in magnitude and did not consistently appear so they were not analyzed further.

The reverse current, Q_4, was not analyzed either because of the difficulty in observing the point of maximum velocity of the dye trace which coincided with the water surface.

2. The Main Inflow Current

The major inflow current at low inflow velocities was Q_1. The pertinent independent variables involved in establishing a dimensionless correlation between the current depth, h_1, the maximum velocity, $\overline{V}_{1\,max}$, and the inflow characteristics are:

$$h_1 = f\left(\rho_{in} - \rho_o, V_{in}, g, \nu, \frac{\Delta\rho}{\Delta y}, D, b_{in}, d_{in}\right)$$

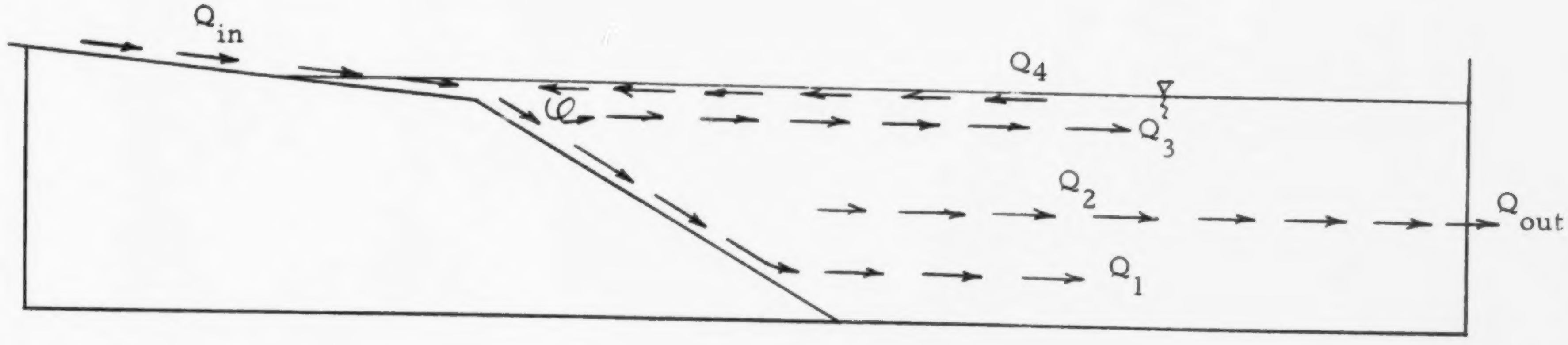

Q_{in}
Q_4
Q_3
Q_2
Q_{out}
Q_1

Figure 17. General current pattern.

and $$\overline{V}_{1\,max} = f\left(\rho_{in} - \rho_o, \frac{\Delta\rho}{\Delta y}, g, Q_{out}, b_{in}, h_1, D, h_{out}, b\right).$$

Using normalizing techniques, the dependent variables may be made dimensionless and written as a function of a number of dimensionless groupings involving the independent variables:

$$Y_1(h_1) = \phi\left(\alpha_1^{A_1}, \alpha_2^{B_1}, \alpha_3^{C_1}, \ldots \alpha_{n-r}^{X_1}\right),$$

and $$Y_2(\overline{V}_{1\,max}) = \phi\left(\beta_1^{A_2}, \beta_2^{B_2}, \beta_3^{C_2}, \ldots \beta_{n-r}^{X_2}\right),$$

but there are several dimensionless groups involving h_1, and $\overline{V}_{1\,max}$, and consequently many different possible groupings for each α and β. Also, since $\alpha_1 \ldots \alpha_{n-r}$ and $\beta_1 \ldots \beta_{n-r}$ are dimensionless, they may group with each other in any possible combination. However, from experience and consideration of the type of variables involved, functional relationships would be expected to be influenced largely by the following criteria:

$Re = \frac{VL}{\nu}$, a form of Reynolds number;

$Fr = \frac{V}{(gh)^{1/2}}$, a form of Froude number;

$\frac{a}{b}$, a geometric ratio;

$\frac{\rho_{max} - \rho_o}{\rho_{max}}$, a density ratio.

The maximum velocity, $\overline{V}_{1\,max}$, of the inflow current, Q_1, was plotted in the form of a Reynolds number against the streamflow Reynolds number in Figure 18. From this plot a relationship is seen between the two parameters, but it varies parametrically with density. Also a reinforcement of Q_1 by the withdrawal current was noticed for an inflow density $\rho_{in} \approx 1.0120$ gr/cm^3. A density scaling factor in the form of $\frac{D}{D-h_1}$ was used and the new relationship is shown in Figure 19. The plot shows that:

$$\frac{\overline{V}_{1\,max} b}{\nu_{res}} = \phi\left(\frac{V_{in} b_{in}}{\nu_{in}} , \frac{D}{D-h_1}\right) .$$

The above relationship was plotted on a semilogarithmic scale (Figure 20). The range of data obtained is nearly monotonical and fit by a straight line on this plot for a large range of $\frac{V_{in} b_{in}}{\nu_{in}} \left(\frac{D}{D-h_1}\right)$. The relationship for $\overline{V}_{1\,max}$ for

$$3000 < \frac{V_{in} b_{in}}{\nu_{in}} < \frac{V_{in} b_{in}}{\nu_{in}} \text{ critical}$$

is as follows:

$$\overline{V}_{1\,max} = \frac{\nu_{res}}{b} \left[-0.5 \text{ Log} \left[\left(\frac{V_{in} b_{in}}{\nu_{in}}\right) \left(\frac{D}{D-h_1}\right) \right] + 365 \right] .$$

It is apparent that as the streamflow velocity is increased,

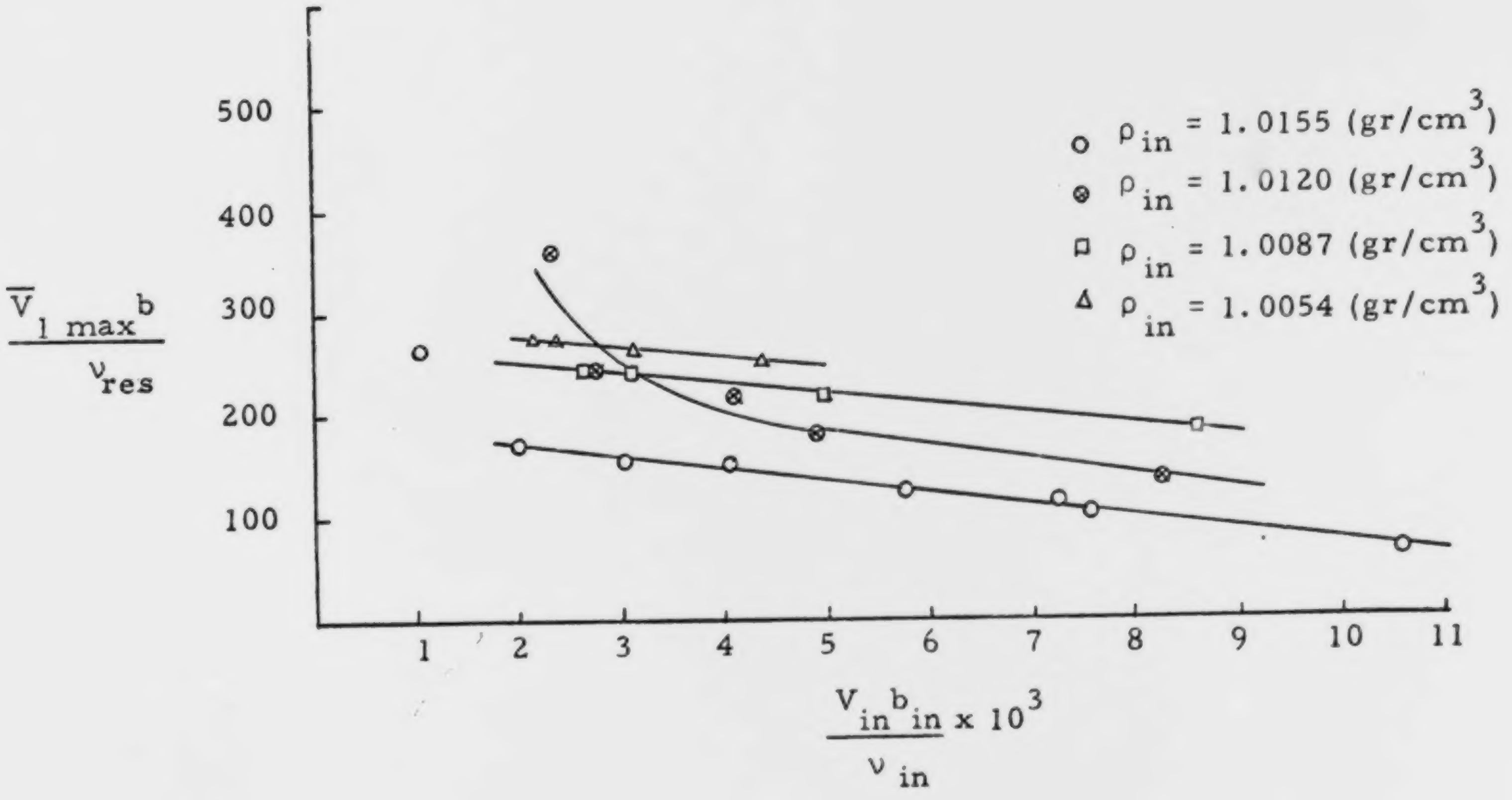


Figure 18. Streamflow Reynolds numbers versus Reynolds numbers of Q_1.

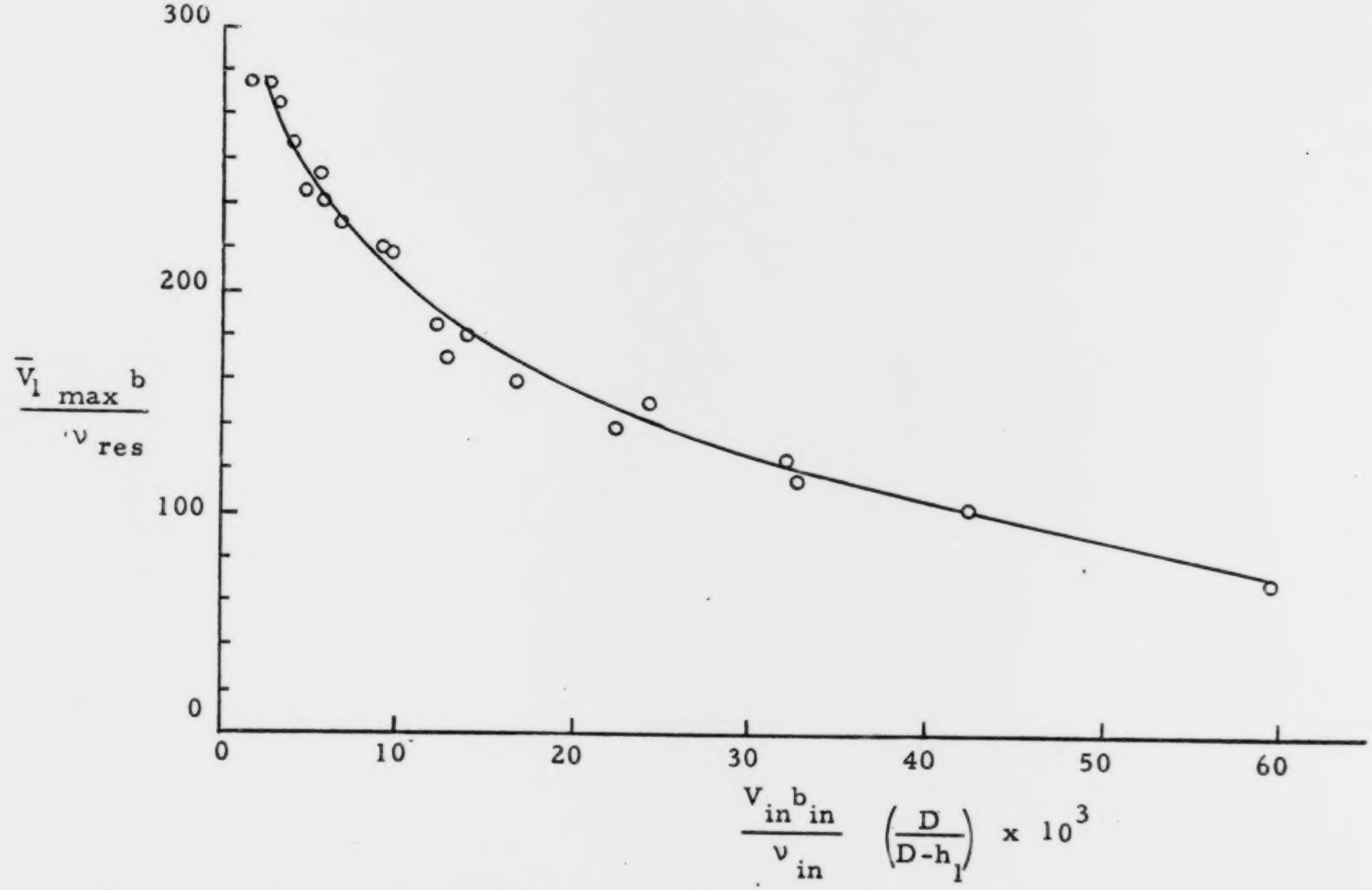


Figure 19. Modified streamflow Reynolds number versus Reynolds number of Q_1.

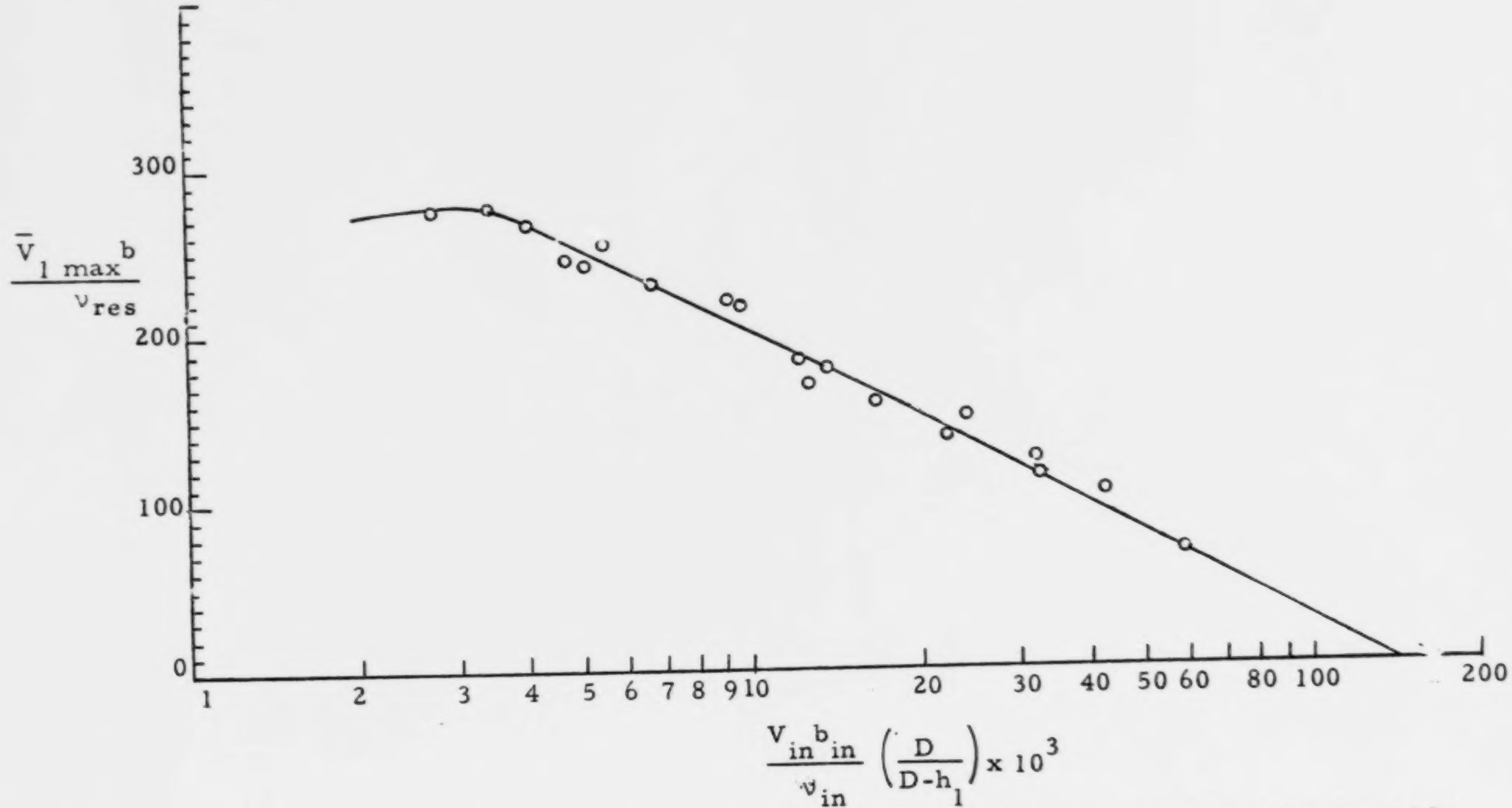


Figure 20. Logarithmic plot of scaled streamflow Reynolds number versus the Reynolds number of Q_1.

the magnitude of Q_1 is reduced, and by extrapolating the curve to $\overline{V}_{1\ max} = 0$ a critical Reynolds number for the existence of Q_1 may be evaluated:

$$(Re)_{crit} = \left(\frac{V_{in} b_{in}}{\nu_{in}}\right)_{critical} = 1.50\text{x}10^5 \left(\frac{D-h_1}{D}\right)$$

At lower values of streamflow velocity the magnitude of Q_1 is seen to reach a maximum value, but complete understanding was not obtained because the nature of the model would not permit,

$$\left(\frac{D}{D-h_1}\right) \frac{V_{in} b_{in}}{\nu_{in}} < 2000.$$

The correct form of the relationship for h_1 was found to be:

$$\frac{h_1}{D} = \phi\left(\frac{V_{in} b_{in}}{\nu_{in}} , \frac{\rho_{in} - \rho_o}{D} \frac{\Delta y}{\Delta \rho}\right).$$

The dimensionless depth, $\frac{h_1}{D}$, was dependent upon $\frac{V_{in} b_{in}}{\nu_{in}}$ only in that for $\frac{V_{in} b_{in}}{\nu_{in}} > \left(\frac{V_{in} b_{in}}{\nu_{in}}\right)_{critical}$, the current, Q_1, did not exist. Figure 21 is a dimensionless plot of the depth current, Q_1, versus a density parameter for $\frac{V_{in} b_{in}}{\nu_{in}} < \left(\frac{V_{in} b_{in}}{\nu_{in}}\right)_{critical}$.

The plot also shows data from Spurkland's (28) work with an underwater diffuser discharging dense fluid into a stratified reservoir. The difference in the relationships is due to the increased mixing

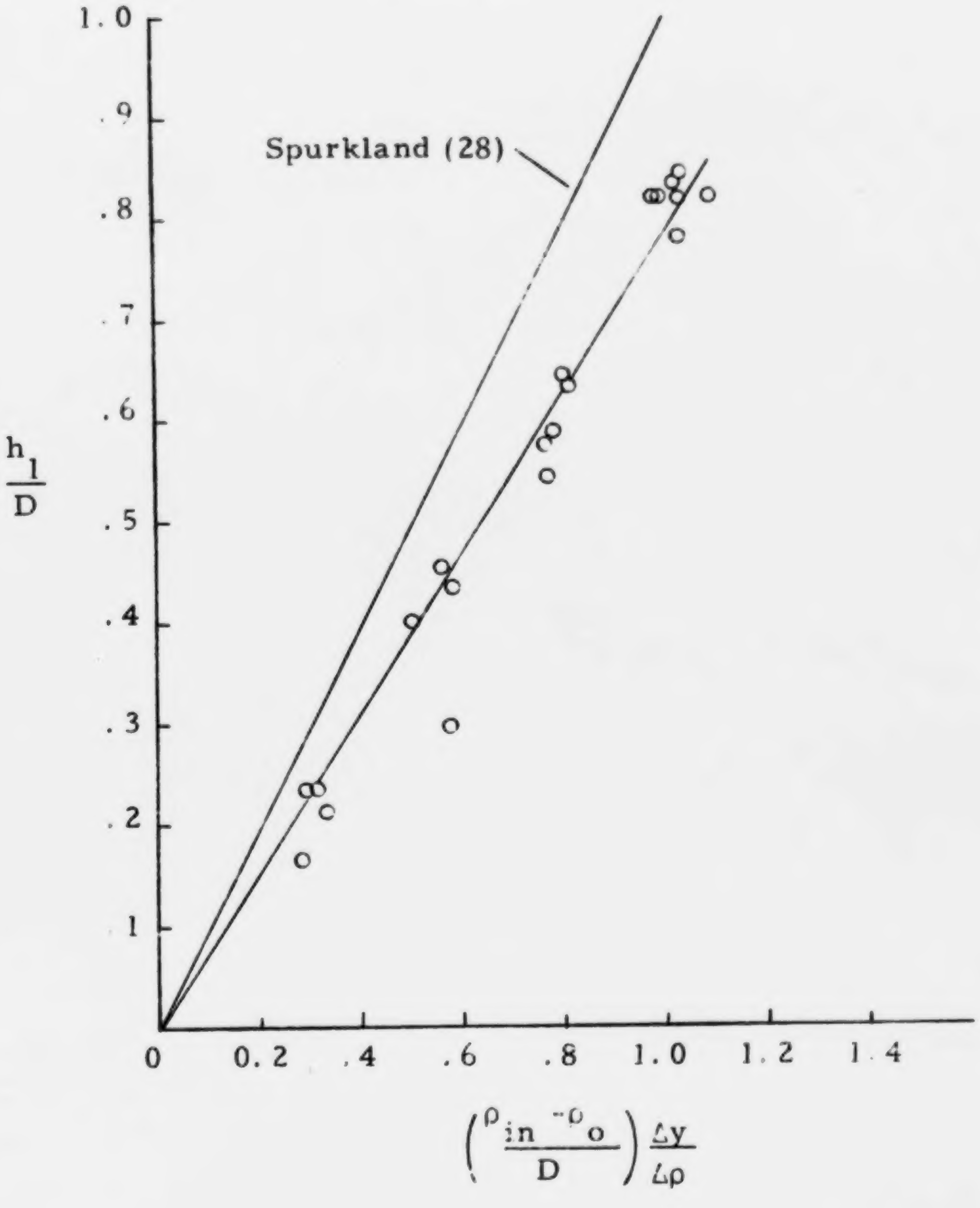


Figure 21. Depth of Q_1 versus density parameter.

associated with the entering streamflow passing through the free surface which lessens the density of the inflow.

3. The Mixing Current

The major reservoir current at high streamflow velocity was the mixing current, Q_3. The pertinent independent variables involved in establishing the inflow-current relationship are similar to those in the previous section,

$$h_3 = f(h_{in}, Q_{in}, V_{in}, \rho_{in}, S, \frac{\Delta\rho}{\Delta y}, \nu_{in}, g, \rho_o),$$

and

$$\overline{V}_{3\ max} = f(Q_{in}, V_{in}, S, \frac{\Delta\rho}{\Delta y}, \nu_{in}, g, \rho_{in}, \rho_o, D, b_{in}).$$

The maximum velocity of the mixing current, Q_3, was found to be independent of the density of the incoming fluid. Figure 22 is a dimensionless plot of the mixing current, densimetric Froude number versus the streamflow Reynolds number. The plot shows that the relationship is linear through a large range of data, but at low values of $\frac{V_{in} b_{in}}{\nu_{in}}$ it verifies a disappearance of Q_3. Unfortunately, insufficient data could be obtained in the region to establish a criterion for the initiation of the mixing current. However, a linear relationship may be provided for a limited range of streamflow Reynolds numbers. The relationship (Figure 22) is

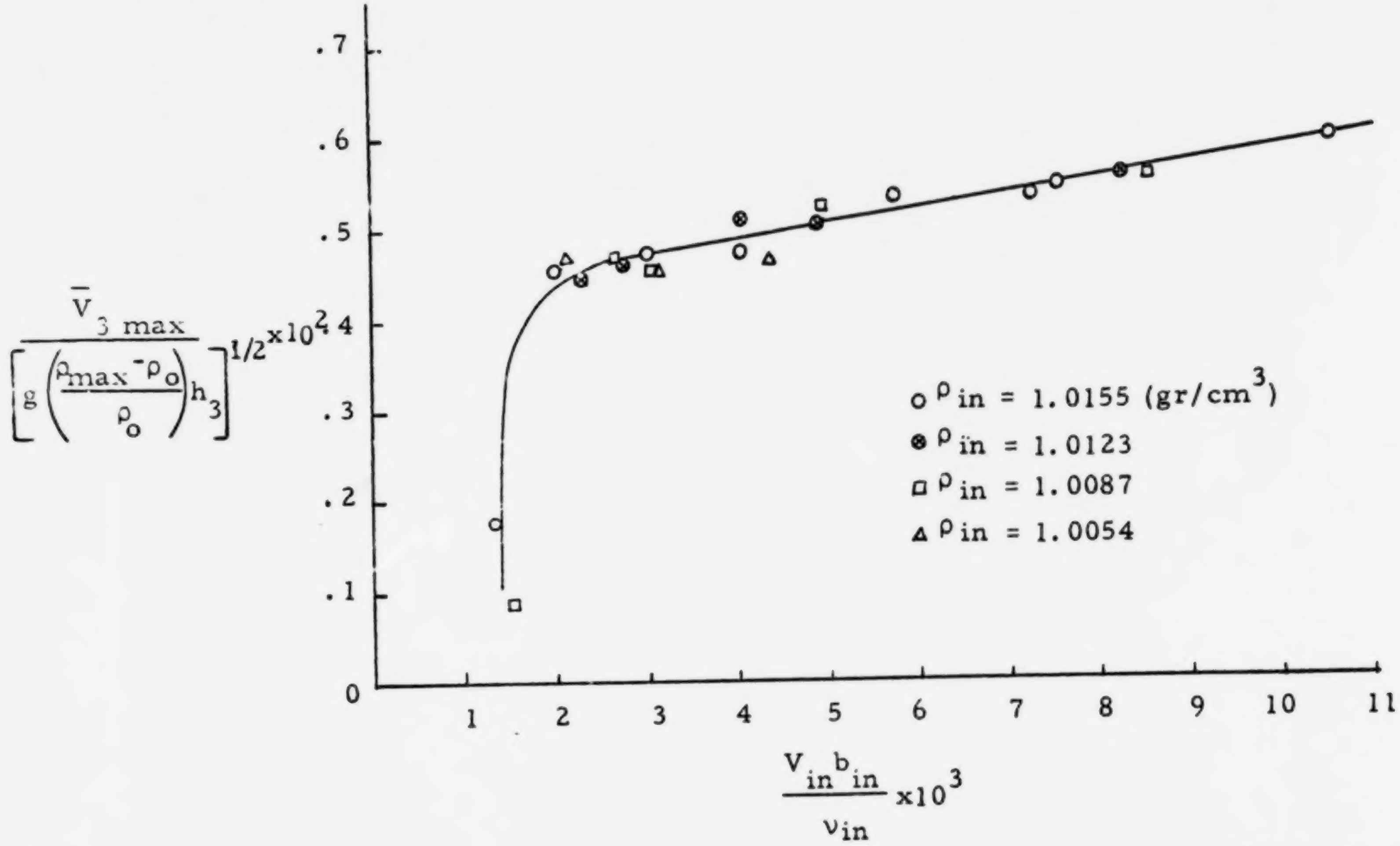


Figure 22. Densimetric Froude No. of Q_3 versus streamflow Reynolds numbers.

$$\overline{V}_{3\,max} = \left[\left(\frac{\rho_{max}-\rho_o}{\rho_{max}}\right) h_3 g\right]^{1/2} \left[1.67 \times 10^{-4} \frac{V_{in} b_{in}}{\nu_{in}} + 0.42\right],$$

for

$$2000 < \frac{V_{in} b_{in}}{\nu_{in}} < 11,000$$

It was expected that the depth, h_3, of Q_3 would follow a relationship of the following form:

$$\frac{h_3}{D} = \phi\left(\frac{V_{in} b_{in}}{\nu_{in}}, S, \frac{\rho_{in}-\rho_o}{D}, \frac{\Delta y}{\Delta \rho}, f(Q_{in}), \frac{h_{in}}{D}\right),$$

but it is shown in Figure 23 that the depth of the mixing current, h_3/D, was independent of all varied independent variables. From this behavior, it must be concluded that h_3/D must be a function of variables held constant in this study or

$$\frac{h_3}{D} = \phi(Q_{in}, h_{in}) .$$

4. The Withdrawal Current

The withdrawal of water from the model reservoir, although intended to be a simplifying step by maintaining a constant water surface elevation during the duration of the experimental run, created a withdrawal current at the elevation of the outlet which extended up the length of the model reservoir. The outlet level was placed about mid-depth in the reservoir and held constant in order to distinguish the effect of the withdrawl current, Q_2, as shown in Figure 24. The figure shows a dimensionless plot of the difference in elevation of Q_1 and Q_2

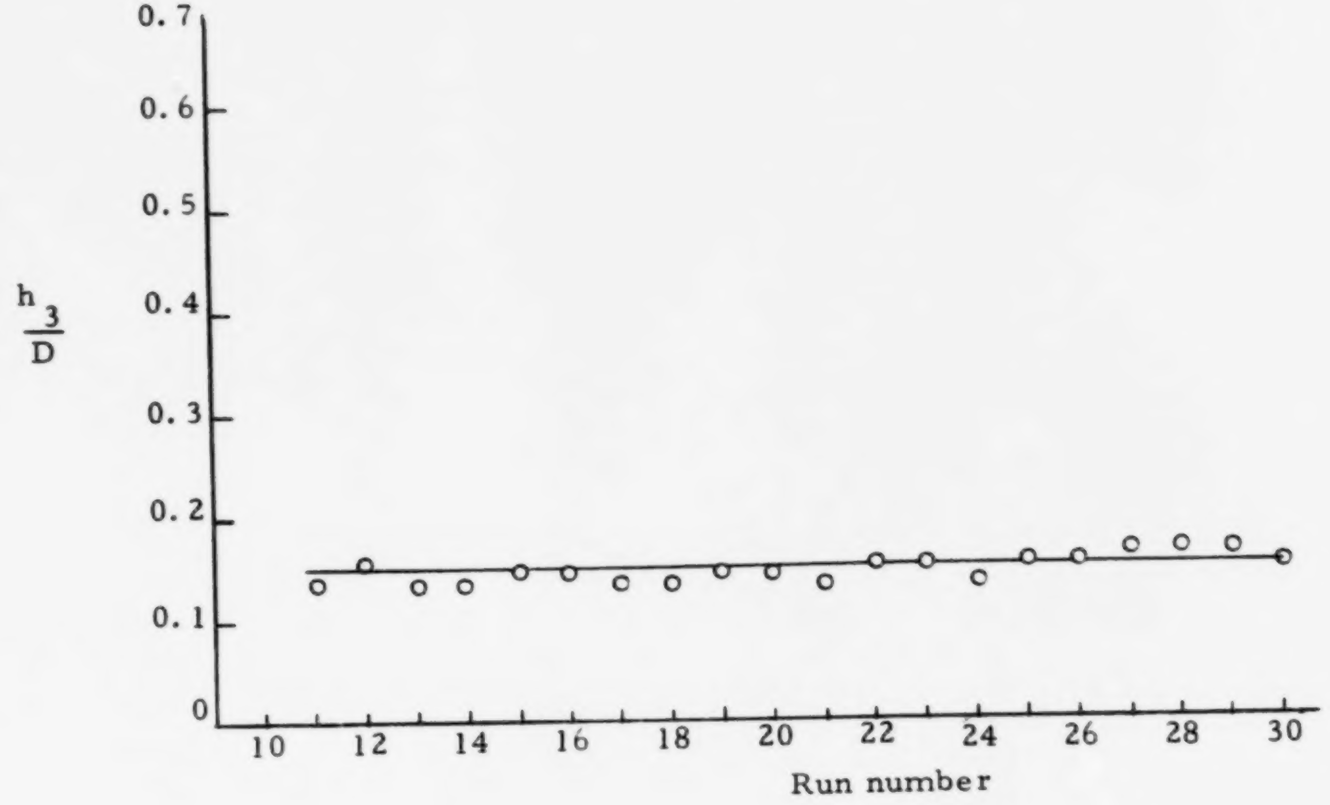


Figure 23. Depth of Q_3 versus experimental run number.

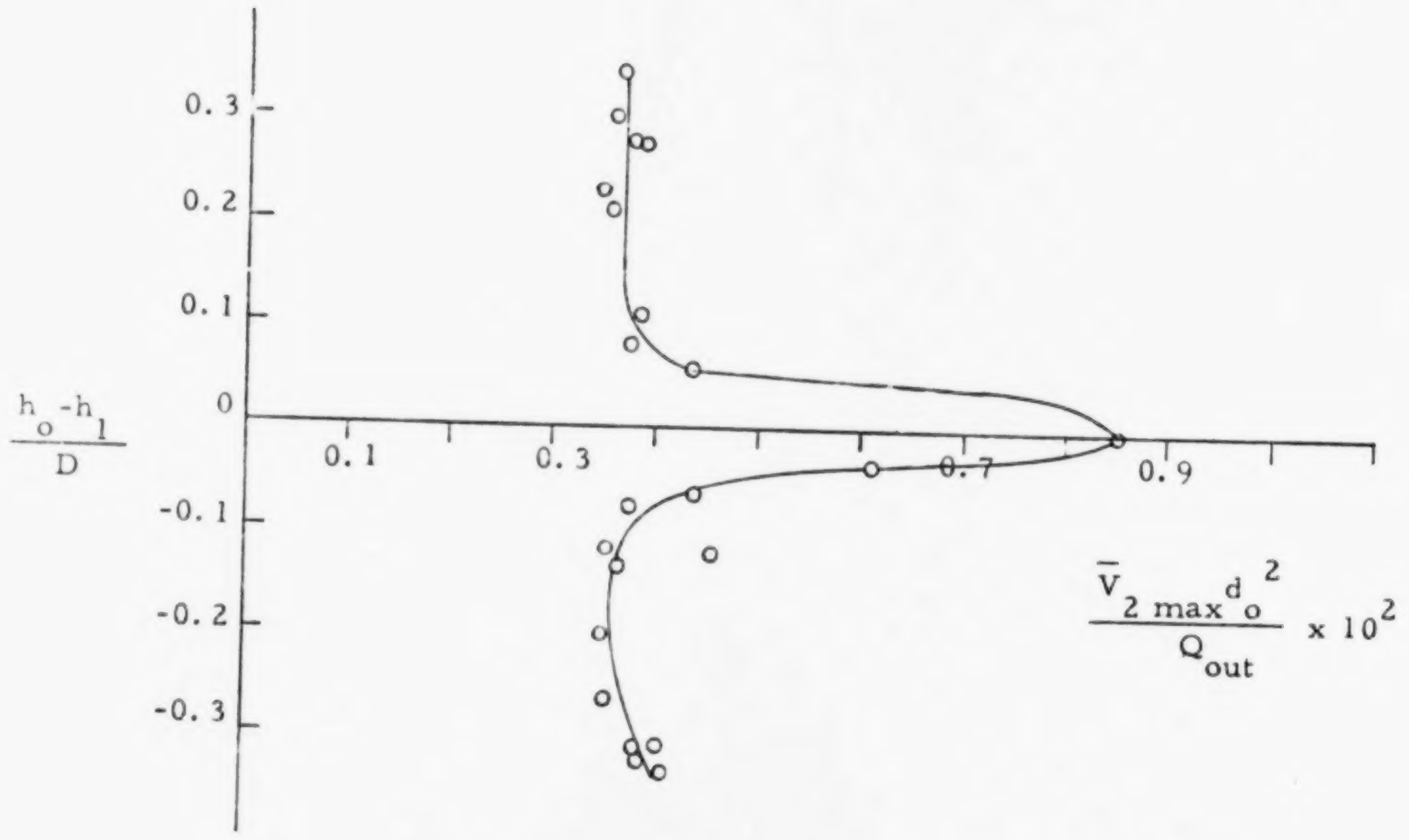


Figure 24. Difference in elevation between Q_1 and the reservoir outlet versus the velocity of Q_2.

versus the maximum velocity, V_2, of the withdrawal current for a constant Q_{in} and Q_{out}. The reinforcing action of the combined Q_1 and Q_2 is easily seen. The maximum reinforcing effect gave the combined current a velocity of two and one-half times the magnitude of the withdrawal current without any reinforcement.

5. Blocking

If the tests were continued for long times, the influence of the length of the tank on the flow was noticed as a blocking phenomena. As the currents approached the end of the tank, their forward movement was impeded. In the case of Q_1, when ρ_{in} was large enough for $h_1 > h_2$, blocking caused the withdrawal current to select entering streamflow as shown in Figure 25.

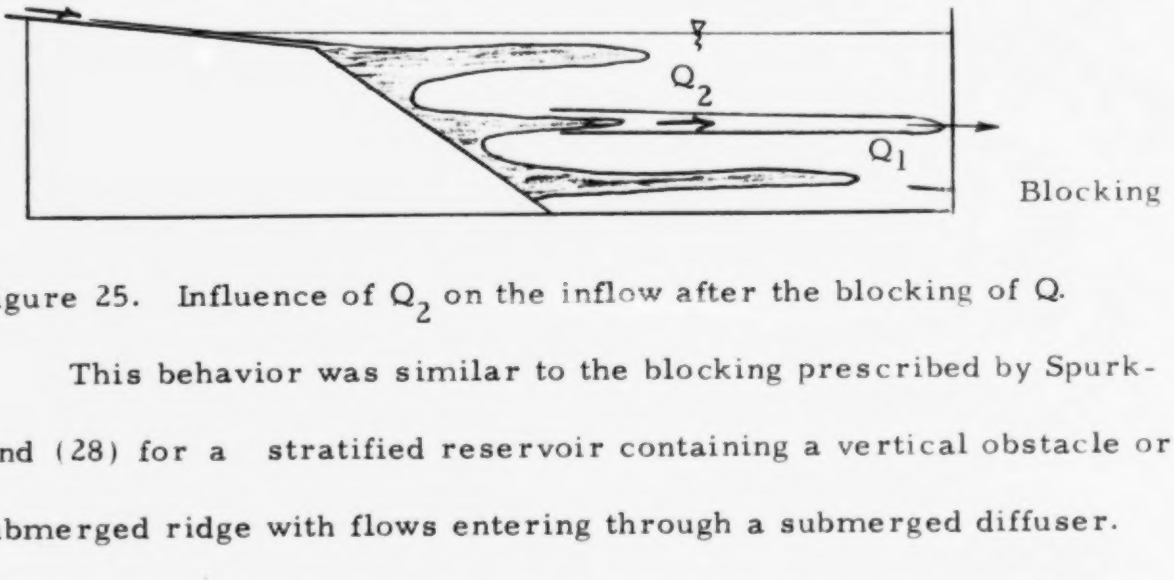


Figure 25. Influence of Q_2 on the inflow after the blocking of Q.

This behavior was similar to the blocking prescribed by Spurkland (28) for a stratified reservoir containing a vertical obstacle or submerged ridge with flows entering through a submerged diffuser. There the main inflow approached a barrier or obstruction and was blocked; with discharge at the barrier boundary another current was created that carried part of the main inflow past the obstruction.

V. DISCUSSION OF THE RESULTS

Some of the effects of entering streamflow on the currents of a density stratified model reservoir were demonstrated in the previous chapter. Correlations between the entering streamflow and the resulting reservoir currents were detailed and some critical parameters established.

In this chapter discussions of errors involved in measurement of the various quantities; limitations present in the investigation; model-prototype relationships; and suggestions for further study will be presented.

1. Summary of Experimental Errors

It is generally realized that errors will be present in making any type of measurement. The probable error present in measuring flow rates, velocities, densities, viscosities, and depths in this study can be estimated as follows in Table 1. The allowable tolerances for the flow rates and length parameters were estimated from the rotameter scale and the various length scales used, while the tolerance for the average streamflow velocity was estimated from the frame speed of the movie camera.

It was first thought that variation in temperature or salt concentration might induce considerable variation in the density or

Table 1. Allowable tolerances in experimental measurements

Tolerance	Units	Magnitude of average measurement
$Q_{in} \pm 0.315$	(cm^3/sec)	12.6
$V_{in} \pm 0.86$	(cm/sec)	30.0
$\overline{V}_{1\ max} \pm 0.002$	(cm/sec)	0.05
$\rho_{in} \pm 0.0005$	(gr/cm^3)	1.0070
$\nu \pm 3\text{x}10^{-6}$	(cm^2/sec	$1.2\text{x}10^{-2}$
$D \pm 0.1$	(cm)	45
$h_1 \pm 0.5$	(cm)	23

viscosity measurements, respectively, but after examining the variation of temperature within the model reservoir (Figure 26) and the difference between reservoir temperatures and calibration temperatures, it was concluded that temperature was negligible in controlling densities. It was also determined that the concentrations of salt solution used had a very minor effect on viscosity.

In the measurements of the reservoir currents by means of dye profiles, the steps involved the projection of slides into a viewer cabinet. In doing so, the various images were first aligned with reference bolts on the front side of the reservoir tank in order to match the scale on the viewer cabinet. Moreover, the distance in from the wall to various dye streaks was slightly

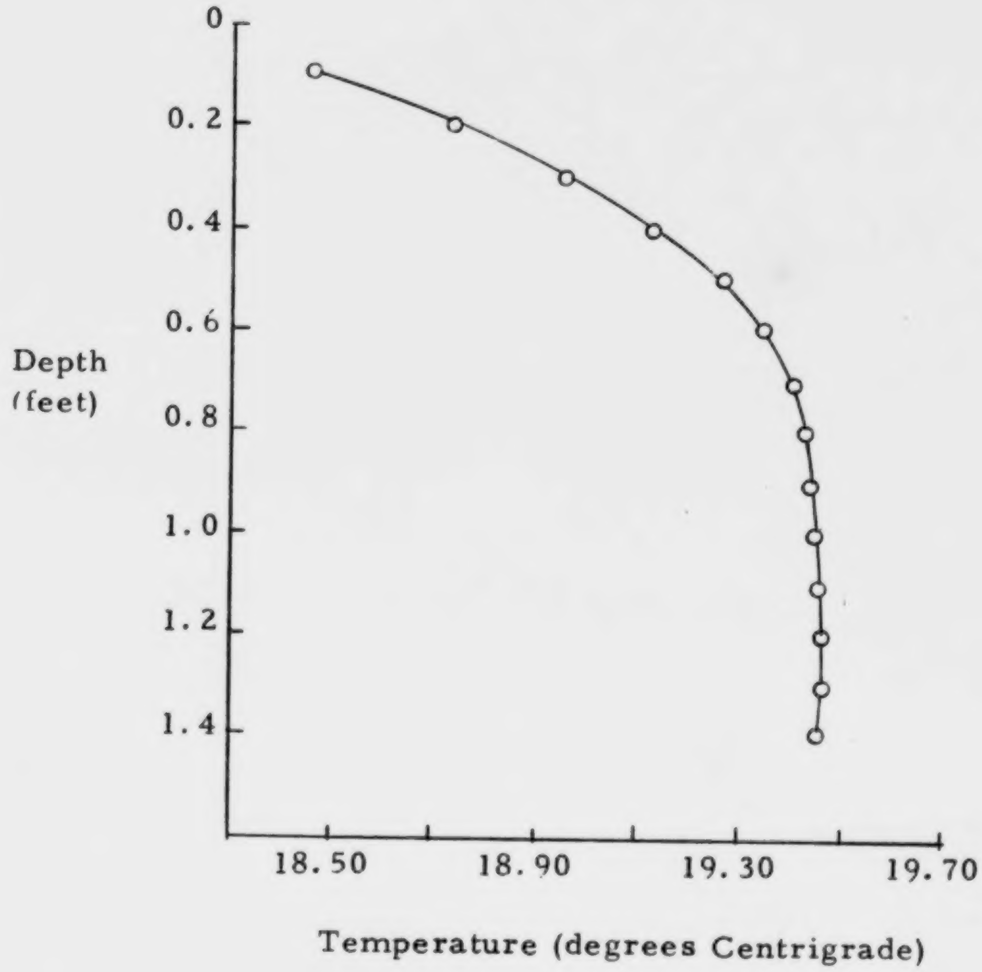


Figure 26. Initial variation of temperature within model reservoir.

variable making slight parallax errors in the photographically determined lengths. Thus, possibly the largest inherent error in taking any measurement occurred in the determination of the reservoir current velocities.

It is believed that the propagation of the above tolerances in computing the parameters plotted in Chapter IV are the cause of much of the scatter shown in Figures 18 through 24.

2. Limitations of the Investigation

Certain assumptions necessary to simplify the analysis in this model study imposed limitations on the results obtained. The streamflow rate, Q_{in}, definitely varies with time in a prototype situation and would be expected to have a large effect in reservoir density current flows. In this study the streamflow rate was held constant. It was seen in the discussion of thermal stratification that the density gradient varies with time and usually also changes with depth. The density gradient was also made constant. The effects of holding the streamflow rate and density gradient constant limits the results considerably. The existence of the h_{in} variable is also limiting in that h_{in}'s meaning should be questioned. Figure 27 shows cross sections of the model configuration and an idealized reservoir.

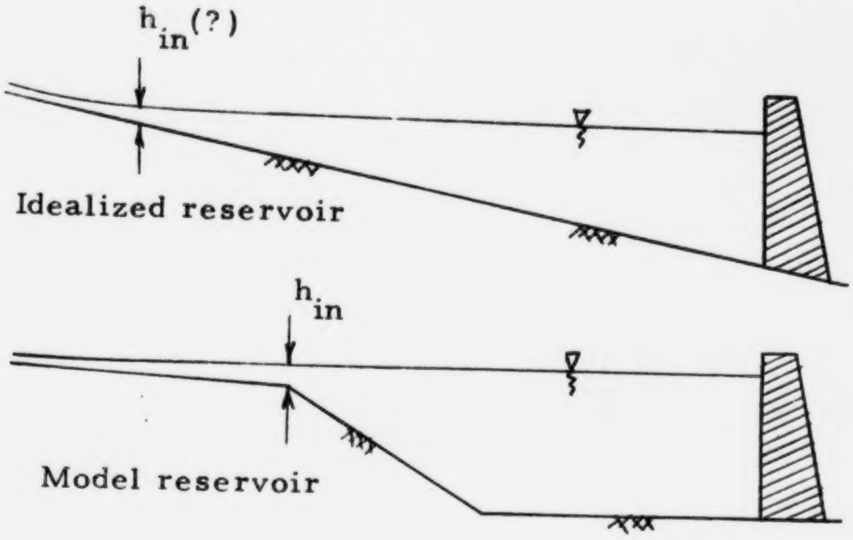


Figure 27. Configuration of an idealized reservoir and the model reservoir.

In the model reservoir a double slope configuration is necessary to insure correctly scaled streamflow velocities while at the same time providing adequate depth in the model reservoir. The depth of water at the intersection of the two slopes is defined as h_{in}. An idealized reservoir is usually described with the bottom of the reservoir and the streambed as one slope, and h_{in} is not really defined, although in some cases sediment may alter the configuration, creating a type of h_{in} parameter.

Time influenced the behavior of the model reservoir currents in many ways. As the inflow currents approached the outlet of the tank, their speed of advancement slowed down due to a blocking phenomena, and the inflow current velocities became a function of time. Secondly, a noticeable shift in the density profile appeared after a period of time due to the combination of withdrawal and inflow in a model reservoir of limited size. Figure 28 shows the density profile both before and after a typical run. Both of these effects were to be disregarded by making two restrictions on the investigation. The experimental data was taken at a reference station which was 10.5 feet from the model stream mouth, and the measurements were not taken beyond the time that blocking has no influence. These restrictions limited the study to be valid only for density flows in the upper reaches of a reservoir. This one reference station also prevented the results from including the effects of variation in x.

Although the flow in the model reservoir was intended to be two-dimensional, variations from two-dimensional flow were observed in the reservoir currents as a meandering from side to side as shown in Figure 29. The meandering presented difficulties in the measurement of the actual reservoir currents because from the side view, the currents appeared to vary in velocity with time. The problem was solved by averaging the photographed current velocities, $V_{i\ max}$, to obtain a net average velocity of advancement, $\overline{V}_{i\ max}$. The meandering phenomena appeared to be a function of the tank geometry and current velocity, and possibly the behavior could be described in terms of a

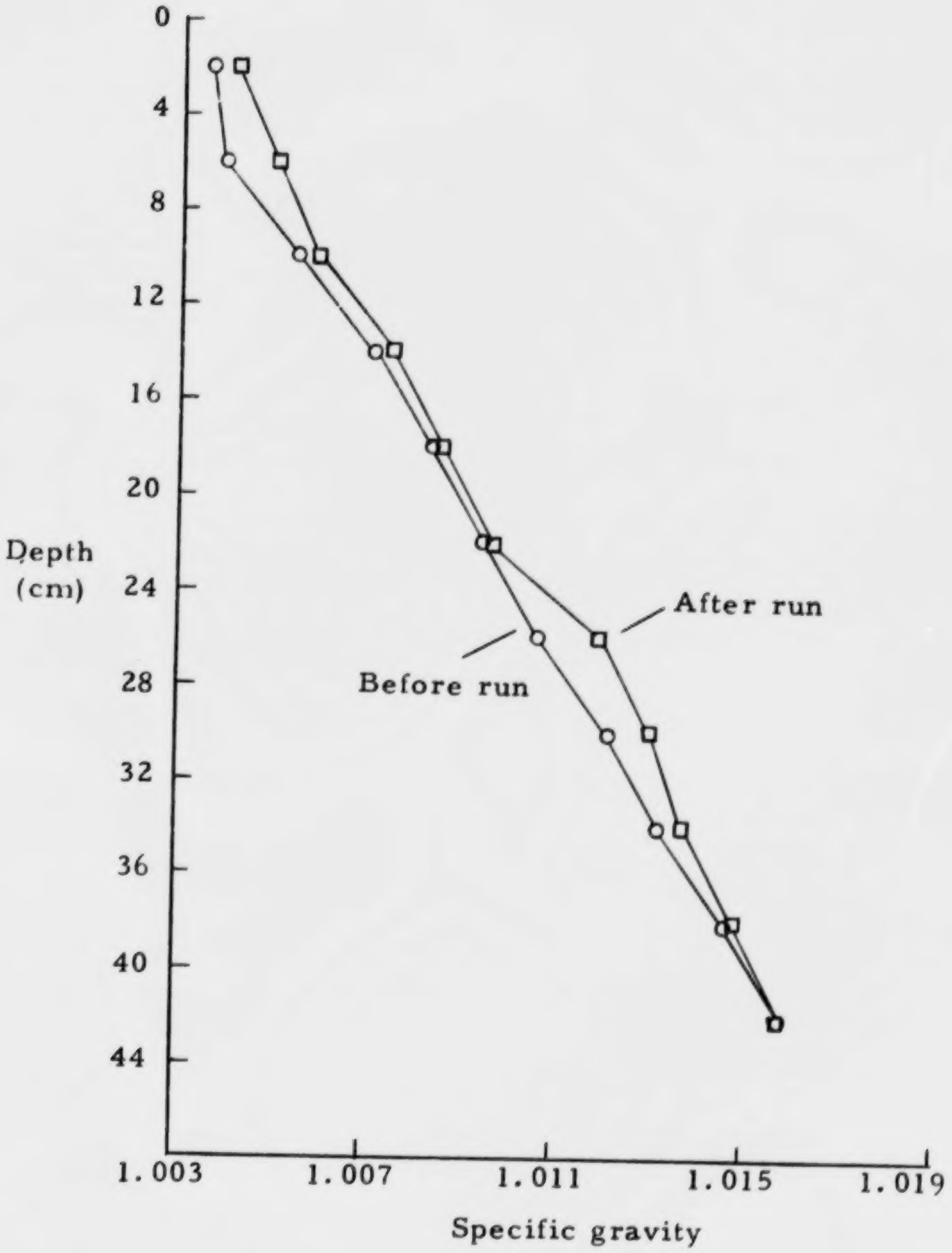


Figure 28. Density profile shift.

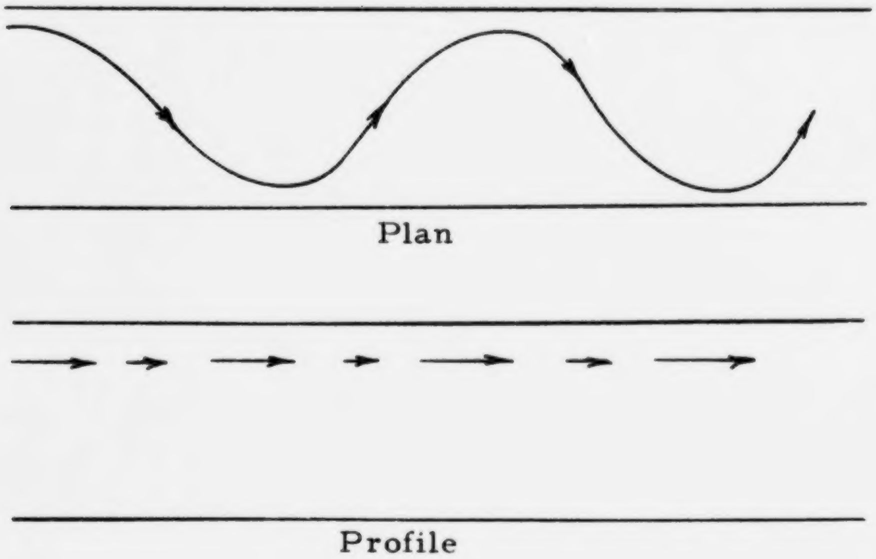


Figure 29. Meandering of reservoir currents.

Strouhal number or Brunt - Väisälä frequency.

3. Model - Prototype Relationship

The scaling of results obtained from a model study to a prototype is based on the laws of similitude, which require the model and the prototype to be similar geometrically, kinematically, and dynamically. Geometric similarity implied that all significant geometric parameters, in dimensionless form, are the same for the model and prototype, and kinematic similarity exists when the streamline patterns in the model and the prototype are the same. Dynamic similarity exists when the ratios of forces at corresponding points in the flow have equal values in both model and prototype and implies both geometric and kinematic similarity.

The requirement for dynamic similar fluid motions of any incompressible viscous free surface fluid in a gravity field is equality of Froude and equality of Reynolds numbers in both systems. Specifying the equality of the Froude numbers,

$$\frac{Fr_m}{Fr_p} = Fr_r = \frac{V_r}{\sqrt{g_r L_r}} = 1.0$$

or

$$V_r = \sqrt{g_r L_r} .$$

From the equality of Reynolds numbers,

$$V_r = \frac{\mu r}{\rho_r L_r} .$$

Since the velocity ratios must be the same, and since for terrestrial events gr=1,

$$L_r = \left(\frac{\mu r}{\rho r}\right)^{2/3} = \nu^{2/3}.$$

For dynamic similitude of both viscous and gravity effects, the choice of fluid determines the length ratio, and since similar fluids are used in the model and the prototype, the criteria cannot be satisfied unless the scale ratio is close to unity. Usually in open channel systems, if the viscous effects are small in comparison to gravity effects, only a Froude number similarity is required.

Using a Froude number scaling criteria and a length ratio, $\frac{L_m}{L_p} = \frac{1}{200}$, Table 2 is formed. Table 2 shows the model-prototype scaling parameters in this investigation.

Table 2. Model-prototype scaling parameters

	Model	Ratio	Prototype
Reservoir:			
length (ft)	20.0	$Lr=5\times10^{-3}$	4000
width (ft)	1.50	$Lr=5\times10^{-3}$	300
depth (ft)	1.48	$Lr=5\times10^{-3}$	296
surface area (ft^2)	30	$AR=2.5\times10^{-5}$	1.2×10^{6}
volume (ft^3)	44.4	$V=1.25\times10^{-7}$	3.55×10^{8}
Stream:			
depth (ft)	0.01	$Lr=5\times10^{-3}$	2
width (ft)	0.046	$Lr-5\times10^{-3}$	29.2
velocity (ft/sec)	0.1-0.9	$Vr=0.0708$	1.41-12.7
discharge (ft^3/sec)	4.46×10^{-4}	$Qr=1.77\times10^{-6}$	252

The Froude scaling assumption requires that the model is large enough to ignore viscous effects. In the experimental runs, however, it appeared that the model reservoir currents behaved as laminar flow, meaning that viscous effects were significant.

How can laminar flow in a modeling scheme provide insight into flows in a prototype reservoir, which are expected to be turbulent because of the large scale or large Reynolds numbers, and how does a model using a Froude scale criteria compare with the prototype reservoir? Consider the inertia forces and resistance forces in the form of a Reynolds number with eddy viscosity, E, included in the resistance term.

$$Re^{I} = \frac{\text{inertia}}{\nu+E}$$

Similarity between the laminar model currents and prototype currents should occur if

$$\left(\frac{\text{inertia}}{\nu+E}\right)_{\text{model}} \approx \left(\frac{\text{inertia}}{\nu+E}\right)_{\text{prototype}}$$

Since the model is laminar in behavior, the eddy viscosity of the model is assumed to be zero. Similarity will be established if $E_{\text{prototype}}$ can be of an order of magnitude to equalize the ratios.

The turbulent eddy viscosity is difficult to quantitize, but an order of magnitude value may be obtained. Assume that reservoir currents due to entering streamflow are a type of columnar flow somewhat similar to a two-dimensional jet. For two-dimensional jetflow, Schlichting (26) has shown that the turbulent eddy viscosity may be expressed as a function of a characteristic velocity, U_{max}, and a length denoting half the width at half depth,

$$E_p = 0.026\ b_{\frac{1}{2}}\ U_{max}.$$

Since it was seen that

$V_m \approx 0.043\ \frac{cm}{sec}$; $b_m = 45$ cm; $E_m = 0$; and

$\nu_m = \nu_p = 1.2 \times 10^{-2}\ cm^2/sec$,

the modified Reynolds numbers are:

$(195)_{model} \approx (78)_{prototype}$.

The Reynolds analogy hypothesis (29), i.e., the eddy diffusion coefficient for mass transport approximates the eddy viscosity coefficient for momentum transport, may also be assumed. Predictions from lake and reservoir measurements by Bella (3) and Orlob (24) have shown effective diffusion coefficients to range from 0.1cm^2/sec to 10cm^2/sec by assuming a one-dimensional assumption with no velocity profile. Expecting the coefficient to be higher where density flows are involved, $E_p = \frac{10cm^2}{sec}$ may be substituted into the prototype modified Reynolds number along with the prototype values for velocity and width,

$(195)_{model} \approx (216)_{prototype}$.

The two ratios of the same order of magnitude suggest that viscosity in the small scale of the model simulates the eddy viscosity in the actual reservoir allowing laminar flow to give insight to prototype reservoir flows. The validity of the Froude scaling could be verified by comparing the characteristics of the model study with characteristics of an actual prototype reservoir, but at this time there is insufficient field evidence.

4. Suggestions for Further Study

A natural extension of this experimental work would be to eliminate a number of limiting assumptions by examining the effect of an increased number of interacting independent variables. Important extensions would involve the variation of the streamflow rate and the density gradient. It would be also important to examine the variation of various factors with the length of the tank and time. An important aspect involving length of the reservoir and time is the blocking effect and meandering. Specifically when and where does blocking occur?

Another phenomena which merits more study is the reinforcing effect between inflowing density currents and withdrawal currents. This phenomenon appears significant in the control of reservoir detention time.

Field data for reservoir density currents is insufficient. Field studies are needed for the verification of laboratory scaling criteria and a greater understanding of the behavior of flow patterns.

VI. SUMMARY AND CONCLUSIONS

An experimental study of entering streamflow effects on currents of a density stratified model reservoir was made. The major conclusions will be summarized as follows:

1. For the range of values tested,the entering model streamflow created two possible main inflow density currents in the model reservoir.

2. The upper inflow current increased its magnitude and the lower inflow current decreased its magnitude as the model streamstreamflow Reynolds number increased. For the range of streamflow parameters tested,these currents could be described by the following relationships:

$$\bar{V}_{1\,max} = \frac{\nu_{res}}{b}\left[-0.5\ \mathrm{Log}\left[\left(\frac{V_{in}b_{in}}{\nu_{in}}\right)\left(\frac{D}{D-h_1}\right)\right] + 365\right],$$

$$\bar{V}_{3\,max} = \left[\left(\frac{\rho_{max}-\rho_o}{\rho_{max}}\right)h_3\,g\right]^{1/2}\left[1.67\times10^{-4}\ \frac{V_{in}b_{in}}{\nu_{in}} + 0.42\right].$$

3. The lower inflow current will no longer occur at a model streamflow number greater than

$$\frac{V_{in}b_{in}}{\nu_{in}} = 1.50\times10^{5}\left(\frac{D-h_1}{D}\right).$$

4. The elevation of the upper inflow current was independent

of V_{in} and ρ_{in}. The elevation of the lower inflow current was dependent on ρ_{in} and the mixing which occurred at the stream mouth.

5. The interaction between two reservoir density currents created a significant reinforcement of both currents.

6. The blocking effect due to reservoir stratification and the influence of geometry may have significant influence on internal model reservoir currents created by entering model streamflow.

7. A reservoir model with laminar behavior probably gives much insight to problems associated with flow in prototype reservoirs.

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APPENDIX A. Summary of Notations.

For simplicity, symbols of secondary importance which are defined in the text are omitted from the following list:

C	Concentration of solute in the stratified fluid
D	Depth of model reservoir
D'	Diffusion coefficient
$\frac{d\rho}{dy}$	Density gradient
E_1	Turbulent eddy coefficient
Fr	Froude number
g	Gravitational acceleration
h_1	Depth from free surface
h_{in}	Depth of change in slope
k	Coefficient of entrainment
P	Pressure
Q_1	General current designation
Re	Modified Reynolds number including a turbulent eddy coefficient
S_v	Slope of streambed
S_r	Slope of upper reservoir floor
s'	Rectangular coordinate in direction of streambed
t	Time
t'	Rectangular coordinate normal to s' vertically

APPENDIX A (continued)

T_1	Temperature
u	Velocity component in x-direction
$u(s', t', v')$	Lagrangian velocity in s-direction
v	Velocity component in y-direction
v'	Rectangular coordinate normal to s' and t'
$V_{1\,max}$	Maximum instantaneous velocity of various reservoir currents
$\overline{V}_{1\,max}$	Maximum average velocity of various reservoir currents
x	Horizontal rectangular coordinate
y	Vertical rectangular coordinate
γ_1	Specific weight
θ	Angle of upper reservoir slope
μ	Kinematic viscosity
ν_1	Dynamic viscosity
ρ_1	Density
$\rho_1(s', v', t')$	Lagrangian density with respect to s, v, t
τ_1	Shearing stress
ϕ	Angle of streambed
ψ	Streamfunction
∇	Gradient operator

APPENDIX A. (continued)

Subscripts

a	Ambient fluid
l	General subscript
in	Inflowing fluid
m	Model
max	Maximum
out	Outflow
p	Prototype
res	Reservoir

APPENDIX B. Values of Physical Constants

D	cm.	45
b_{in}	cm.	4.45
b	cm.	45.7
L	cm.	580.
d_o	cm.	0.952
h_o	cm.	23.0
θ	degrees	9.2
ρ_o	gr/cm^3	1.001
Q_{in}	cm^3/sec	12.6
Q_o	cm^3/sec	12.6
h_{in}	cm.	1.27
g	cm/sec^2	980
ρ_{max}	gr/cm^3	1.017

APPENDIX C. Summary of Data

Test	11	12	13	14	15	16	17
S_v	0.0052	0.0070	0.0070	0.0052	0.0096	0.0096	0.0096
V_{in} (cm/sec)	5.45	6.37	7.28	5.85	8.32	7.47	8.38
ρ_{in}(gr/cm^3)	1.0155	1.0123	1.0087	1.0054	1.0155	1.0120	1.0083
$\frac{\Delta\rho}{\Delta y}$ (gr/cm^4)x10^4	3.13	3.25	2.95	3.16	3.31	3.13	3.24
d_{in} (cm)	0.518	0.445	0.388	0.484	0.340	0.379	0.338
T_{in} (^{o}C)	13.0	12.5	13.0	12.5	12.5	13.5	12.5
ν_{in} (cm^2/sec)x10^2	1.213	1.227	1.213	1.227	1.227	1.199	1.227
$\overline{V}_{1\,max}$(cm/sec)x10^2	4.07	8.48	5.93	6.57	3.81	5.60	5.80
$\overline{V}_{2\,max}$(cm/sec)x10^2	5.72	8.48	6.00	5.21	5.68	5.25	5.34
$\overline{V}_{3\,max}$(cm/sec)x10^2	4.49	4.74	4.66	4.66	4.87	4.75	4.53
h_1 (cm)	38.0	24.5	19.5	10.5	37.0	26.5	18.0

APPENDIX C. (continued)

Test	11	12	13	14	15	16	17
h_2 (cm)	23.0	23.5	23.0	23.0	23.0	23.0	23.0
h_3 (cm)	6.0	7.0	6.0	6.0	6.5	6.5	6.0
T_{res} (°C)	17.0	17.5	16.5	17.0	17.0	17.5	17.0
ν_{res} (cm^2/sec)x10^2	1.093	1.079	1.106	1.093	1.093	1.079	1.093
$\frac{\bar{V}_{1\,max} b}{\nu_{res}}$	170.5	359.0	245.0	275.0	159.5	231.0	242.7
$\frac{V_{in} b_{in}}{\nu_{in}}$	2006.0	2311.0	2670.0	2120.0	3010.0	2770.0	3040.0
$\frac{D}{D-h_1}$	6.43	2.20	1.77	1.30	5.62	2.43	1.67
$\frac{h_1}{D}$	0.845	0.545	0.433	0.233	0.822	0.589	0.400
$\frac{h_3}{D}$	0.133	0.155	0.133	0.133	0.144	0.144	0.133

APPENDIX C. (continued)

Test	11	12	13	14	15	16	17
$\frac{h_o - h_1}{D}$	-0.333	-0.033	-.078	0.278	-0.311	-0.078	0.111
$\frac{\overline{V}_{2\,max} d_o^2}{Q_{out}} \times 10^2$	0.411	0.610	0.434	0.375	0.408	0.377	0.384
$\left(\frac{\rho_{in} - \rho_o}{D}\right) \frac{\Delta y}{\Delta \rho}$	1.032	0.772	0.577	0.309	0.977	0.780	0.500
$\frac{\overline{V}_{3\,max}}{g \frac{\rho_{max} - \rho_0}{\rho_{max}} h_3} \times 10^2$	0.453	0.443	0.470	0.470	0.472	0.460	0.457
$\left(\frac{V_{in} b_{in}}{\nu_{in}}\right) \frac{D}{D - h_1}$	12870.	5080.	4720.	2760.	16900.	6740.	5080.

APPENDIX C. (continued)

Test	18	19	20	21	22	23	24
S_v	0.0096	0.0165	0.0183	0.0165	0.9165	0.0218	0.0209
V_{in} (cm/sec)	8.48	11.19	11.13	13.47	12.20	16.36	13.81
ρ_{in} (gr/cm^3)	1.0054	1.0155	1.0118	1.0087	1.0054	1.0155	1.0120
$\frac{\Delta\rho}{\Delta y}$ (gr/cm^4)x10^4	3.33	3.16	3.13	3.07	3.00	2.96	3.06
d_{in} (cm)	0.334	0.253	0.255	0.210	0.232	0.173	0.206
T_{in} (^{o}C)	13.0	12.5	13.0	13.0	12.0	11.0	11.5
ν_{in} (cm^2/sec)x10^2	1.213	1.227	1.213	1.213	1.242	1.270	1.256
$\overline{V}_{1\ max}$ (cm/sec)x10^2	6.40	3.56	5.21	5.20	6.23	3.18	4.66
$\overline{V}_{2\ max}$ (cm/sec)x10^2	5.38	5.38	6.10	4.36	4.95	5.34	5.08
$\overline{V}_{3\ max}$ (cm/sec)x10^2	4.49	4.87	5.26	5.34	5.00	5.72	5.04
h_1 (cm)	10.5	37.5	26.0	20.5	9.5	37.0	29.0

APPENDIX C. (continued)

Test	18	19	20	21	22	23	24
T_{res} (^{o}C)	17.0	17.5	17.0	16.5	16.0	14.5	14.0
$\nu_{res}(cm^2/sec)x10^2$	1.093	1.079	1.093	1.106	1.120	1.165	1.181
$\frac{\bar{V}_{1\,max}b}{\nu_{res}}$	267.8	151.0	218.1	220.9	254.2	124.8	180.5
$\frac{V_{in}b_{in}}{\nu_{in}}$	3110.0	4050.0	4070.0	4930.0	4360.0	5730.0	4890.0
$\frac{D}{D-h_1}$	1.30	6.00	2.37	1.84	1.27	5.63	2.82
$\frac{h_1}{D}$	0.233	0.834	0.578	0.455	0.211	0.822	0.645
$\frac{h_3}{D}$	0.133	0.144	0.144	0.133	0.155	0.155	0.133
$\frac{h_o-h_1}{D}$	0.278	-0.322	-0.067	0.056	0.300	-0.311	-0.133

APPENDIX C. (continued)

Test	18	19	20	21	22	23	24
$\frac{\overline{V}_{2\,max} d_o^2}{Q_{out}} \times 10^2$	0.387	0.387	0.439	0.313	0.356	0.384	0.365
$\frac{\rho_{in} - \rho_o}{D} \frac{\Delta y}{\Delta \rho}$	0.294	1.021	0.767	0.557	0.326	1.090	0.799
$\frac{\overline{V}_{3\,max}}{\left[g\left(\frac{\rho_{max} - \rho_0}{\rho_{max}}\right)h_3\right]^{\frac{1}{2}}} \times 10^2$	0.452	0.472	0.510	0.525	0.468	0.534	0.509
$\left(\frac{V_{in} b_{in}}{\nu_{in}}\right)\frac{D}{D-h_1}$	4040.	24,300.	9660.	9070.	5540.	32,210.	12,800.

APPENDIX C. (continued)

Test	25	26	27	28	29	30	31
S_v	0.0326	0.0387	0.0383	0.0409	0.0387	0.0622	0.0030
V_{in} (cm/sec)	20.28	21.10	23.03	23.52	22.70	28.83	4.16
ρ_{in}(gr/cm^3)	1.0155	1.0155	1.0120	1.0089	1.0050	1.0151	1.0108
$\frac{\Delta\rho}{\Delta y}$ (gr/cm^4)x10^4	3.16	3.13	3.00	3.06	3.13	3.18	3.11
d_{in} (cm)	0.140	0.134	0.123	0.120	0.125	0.098	0.410
T_{in} (^{o}C)	12.0	12.0	12.0	12.5	12.0	13.0	22.0
ν_{in}(cm^2/sec)x10^2	1.242	1.242	1.242	1.227	1.242	1.213	0.969
$\overline{V}_{1\ max}$(cm/sec)x10^2	2.80	2.54	3.39	1.53	10.21	1.70	11.96
$\overline{V}_{2\ max}$(cm/sec)x10^2	4.91	5.59	6.35	4.91	5.04	5.34	11.96
$\overline{V}_{3\ max}$(cm/sec)x10^2	5.72	5.84	6.14	6.18	10.21	6.42	1.99

APPENDIX C.   (continued)

Test	25	26	27	28	29	30	31
h_1 (cm)	35.0	37.0	28.5	13.5	7.5	37.0	23.0
h_2 (cm)	23.0	23.0	24.0	23.0	23.0	23.5	23.0
h_3(cm)	7.0	7.0	7.5	7.5	7.5	7.0	8.0
T_{res} (°C)	16.0	16.0	16.0	16.0	16.5	16.0	22.5
ν_{res}(cm^2/sec)	1.120	1.120	1.120	1.120	1.106	1.120	0.960
$\frac{\overline{V}_{1\,max} b}{\nu_{res}}$	114.3	103.7	138.4	185.0	422.0	69.4	---
$\frac{V_{in} b_{in}}{\nu_{in}}$	7260.0	755.0	8250.0	8530.0	8130.0	10,580.0	1910.0
$\frac{D}{D-h_1}$	4.50	5.63	2.72	1.43	1.20	5.63	2.05
$\frac{h_1}{D}$	0.778	0.822	0.634	0.300	0.167	0.822	0.510

APPENDIX C. (continued)

Test	25	26	27	28	29	30	31
$\frac{h_3}{D}$	0.155	0.155	0.166	0.166	0.166	0.155	0.178
$\frac{h_0 - h_1}{D}$	-0.266	-0.311	-0.122	0.211	0.344	-0.311	-0.011
$\frac{\overline{V}_{2\,max}\, d_o^2}{Q_{out}} \times 10^2$	0.353	0.402	0.457	0.353	0.362	0.384	0.861
$\frac{\rho_{in} - \rho_o}{D} \frac{\Delta y}{\Delta \rho}$	1.029	1.033	0.814	0.572	0.283	0.986	0.710
$\frac{\overline{V}_{3\,max}}{\left[g\left(\frac{\rho_{max} - \rho_o}{\rho_{max}}\right)h_3\right]^{\frac{1}{2}}} \times 10^2$	0.534	0.546	0.554	0.554	0.921	0.600	0.175
$\left(\frac{V_{in} b_{in}}{\nu_{in}}\right)\frac{D}{D - h_1}$	32,600.	42,500.	22,400	12,200	9750.	59,600.	3920.0

APPENDIX C. (continued)

Test	32	33	34
S_v	0.0021	0.1575	0.0011
V_{in} (cm/sec)	3.52	55.5	2.74
ρ_{in} (gr/cm^3)	1.0087	1.0150	1.0123
$\frac{\Delta\rho}{\Delta y}$ (gr/cm^4)x10^4	3.06	3.12	3.01
d_{in} (cm)	0.423	0.048	0.438
T_{in} (°C)	22.0	18.5	18.5
ν_{in}(cm^2/sec)x10^2	0.969	1.030	1.030
$\overline{V}_{1\ max}$(cm/sec)x10^2	5.76	0.53	6.06
$\overline{V}_{2\ max}$(cm/sec)x10^2	4.53	4.90	4.62
$\overline{V}_{3\ max}$(cm/sec)x10^2	0.98	30.90	1.95

APPENDIX C. (continued)

Test	32	33	34
h_1 (cm)	13.0	34.0	29.0
h_2 (cm)	23.0	23.0	23.0
h_3 (cm)	6.0	7.0	7.5
T_{res} (°C)	22.5	20.0	20.0
ν_{res} (cm^2/sec)	0.960	1.004	1.004
$\frac{\overline{V}_{1\ max} b}{\nu_{res}}$	274.0	24.2	277
$\frac{V_{in} b_{in}}{\nu_{in}}$	1615.0	24,000	1186
$\frac{D}{D-h_1}$	1.41	4.09	2.81
$\frac{h_1}{D}$	0.289	0.756	0.634

APPENDIX C. (continued)

Test	32	33	34
$\frac{h_3}{D}$	0.133	0.155	0.166
$\frac{h_0-h_1}{D}$	0.222	-0.200	-0.133
$\frac{\overline{V}_{2\,max} d_o^2}{Q_{out}} \times 10^2$	0.326	0.353	0.332
$\frac{\rho_{in}-\rho_o}{D} \frac{\Delta y}{\Delta \rho}$	0.572	1.029	0.814
$\frac{\overline{V}_{3\,max}}{g \frac{\rho_{max}-\rho_o}{\rho_{max}} h_3} \times 10^2$	0.099	2.910	0.170
$\frac{V_{in} b_{in}}{\nu_{in}} \frac{D}{D-h_1}$	2280.0	98,300.	3330

PART II. THE NUMAC METHOD FOR NONHOMOGENEOUS UNCONFINED MARKER-AND-CELL CALCULATIONS

Michael D. Terry
Howard T. Mercier
Larry S. Slotta

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ABSTRACT

A marker and cell method for computations involving nonhomogeneous, incompressible, viscous fluids is developed. New boundary conditions which are useful in hydrodynamic and oceanographic simulation are presented. A wide range of applications are included featuring both graphic and numerical computer output. A flow chart and a listing for those interested in implementing the method are included.

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THE NUMAC METHOD FOR NONHOMOGENEOUS UNCONFINED MARKER-AND-CELL CALCULATIONS

I. INTRODUCTION

In the field of fluid mechanics, the governing equations of motion are non-linear partial differential equations. Because of this non-linearity, analytical solutions can be obtained only for highly simplified flow patterns. In order to solve the equations of motion for more sophisticated problems, various numerical methods have been successfully applied. These methods include: (1) reduction to ordinary differential equations so that numerical integration techniques may be used; (2) linearization techniques to reduce the equations to the point where analytical solutions may be obtained; and (3) finite-difference methods to reduce the equations to a set of algebraic equations which are solved by either direct or iterative techniques.

The first two of these methods are limited in application because they are restrictive and involve much detailed analytical work. A stringent restriction placed on fluid problems by these techniques is that of steady flow; i. e., time derivatives of variables must vanish. The third method mentioned above--finite differences--allows the user to solve most types of fluid problems, including those involving unsteady flow.

Fluid flow is generally described using one of the following viewpoints. (1) Eulerian: Attention is focused on some point in space and the changes in the fluid can be described as functions of time at this point. (2) Lagrangian: Attention is focused on an infinitesimal fluid element and the changes in this fluid element can be expressed as functions of time. Major analytical works in fluid dynamics use one or both of these viewpoints; correspondingly numerical techniques have developed along these lines.

The early papers on numerical techniques for fluid problems (Harlow, 1955; Evans and Harlow, 1957) used the Lagrangian viewpoint. Instead of considering every infinitesimal fluid element, attention was focused on a finite number of these elements. By marking the elements being considered, the fluid was conveniently represented by an array of particles. This representation by particles is the primary feature of all Lagrangian numerical techniques; the fluid properties such as density and velocity are localized to a finite number of particles which move with the fluid.

Lagrangian methods have the following advantages. Some parts of the fluid may be resolved more finely than others, fluid interfaces including free surfaces may be precisely defined, and arbitrarily shaped rigid boundaries can be used. On the other hand, large distortions from the initial configuration produce large errors.

Later (Langley, 1959; Welch et al., 1966) Eulerian techniques were developed for fluid problems. Instead of considering the fluid at all spatial points, attention is focused on a finite number of fixed points. Eulerian numerical techniques are characterized by finding the values of the fluid variables at the mesh points of a fixed grid.

Eulerian methods have several useful advantages. The fluid may undergo arbitrarily great distortions without loss of accuracy and outflow walls are particularly easy to handle. However, local resolution is difficult to achieve and interfaces become blurred.

It was shown by Welch et al. (1966) that a system containing two discrete fluids could be handled using a mixed Eulerian-Lagrangian scheme. In this scheme the velocity and pressure were considered as Eulerian variables and found at the mesh points of a fixed grid. The density was considered a Lagrangian variable and was localized to fluid particles.

The method developed by Welch et al. at Los Alamos Scientific Laboratory was called the Marker-And-Cell (MAC) method. It represented a significant advancement in the art of computer simulation of nonhomogeneous, incompressible, viscous fluids.

One of the shortcomings of the original MAC code was its inflexibility in the type of boundary conditions it could handle. For instance, MAC was restricted to inlet velocities that were constant across the inlet and held fixed for the entire run. Such inlets are not useful for finding the transient flow from an "infinite" reservoir whose upstream section is modeled by an inlet.

A second drawback was the consumption of computer time. By adding the technique of overrelaxation, it was found that savings of up to fifty percent could be obtained.

This paper presents the NUMAC, a method for nonhomogeneous unconfined marker-and-cell calculations. The NUMAC is especially useful in oceanographic and hydraulic problems which require an inlet or outlet for modeling regions upstream or downstream from the region of interest. Two types of nonhomogeneities are considered: those involving two immiscible fluids and those involving a single fluid with small local density variations. Examples of both types are included in Chapter IX.

II. EQUATIONS AND BOUNDARY CONDITIONS

To describe the motion of a nonhomogeneous incompressible fluid with constant viscosity it is necessary to determine

velocity, $\vec{w} = u\vec{i} + v\vec{j}$;
density, ρ; and
pressure, P;

as functions of time and position. To find these unknowns Mercier (1968) has shown it is sufficient to solve the equations describing

conservation of mass,
conservation of momentum, and
incompressibility.

These equations are respectively:

The incompressibility equation

$$\frac{\partial\rho}{\partial t} + (\vec{w} \cdot \nabla)\rho = 0 \tag{2. 1}$$

The continuity equation

$$\nabla \cdot \vec{w} = 0 \tag{2. 2}$$

The equation of motion for laminar viscous flow, commonly known as the Navier-Stokes equation

$$\rho \frac{\partial \vec{w}}{\partial t} + \rho(\vec{w} \cdot \nabla)\vec{w} = \rho \vec{g} - \nabla P + 2(\nabla \cdot \mu \nabla)\vec{w} + \nabla x (\mu \nabla x \vec{w}), \qquad (2.3)$$

$\vec{g}$ being the gravitational forces per unit volume and μ the viscosity.

When solving equations numerically, it is frequently desirable that the variables be nondimensionalized and have magnitudes less than unity. Equations (2. 1)-(2. 3) can be scaled by the transformation of variables

$$x = Lx'$$

$$y = Ly'$$

$$t = \frac{L}{W} t'$$

$$\vec{w} = W \vec{w}'$$

$$\rho = R\rho'$$

$$P = RW^2 P'$$

By defining the operator

$$\nabla' = \frac{\partial}{\partial x'} + \frac{\partial}{\partial y'}$$

Equations (2. 1), (2. 2), (2. 3) become respectively

$$\frac{\partial \rho'}{\partial t'} + (w' \cdot \nabla')\rho' = 0.$$

$$\nabla' \cdot \vec{w}' = 0,$$

$$\rho' \frac{\partial \vec{w}'}{\partial t'} + \rho'(\vec{w}' \cdot \nabla')\vec{w}' = \rho' \vec{g}' - \nabla' P' + 2(\nabla' \cdot \mu' \nabla')\vec{w} + \nabla' x(\mu' \nabla' x \vec{w})$$

where $\vec{g}\,' = L\vec{g}/W^2$ and $\mu' = \mu/LRW$. Thus the equations to be solved have the same form before and after scaling. Hereafter, it will be assumed that the equations have been scaled appropriately.

Boundary Conditions

In addition to the equations of motion, boundary and initial conditions must be satisfied. There are usually free surface and material boundary conditions. It is frequently desirable to study some small portion of a larger flow. Consequently, inflow and outflow boundary conditions are also considered.

Let $s(x, y, t) = 0$ be the entire fluid surface. In general s may contain material boundaries, free surfaces, inlets and outlets. The unit vector normal to s is defined

$$\vec{n} = \frac{\nabla s}{|\nabla s|} = \frac{\frac{\partial s}{\partial x}\vec{i} + \frac{\partial s}{\partial y}\vec{j}}{\sqrt{(\frac{\partial s}{\partial x})^2 + (\frac{\partial s}{\partial y})^2}} .$$

Thus, we may express $\vec{n}$ as

$$\vec{n} = n_x \vec{i} + n_y \vec{j} \tag{2.4}$$

where

$$n_x = \frac{\frac{\partial s}{\partial x}}{|\nabla s|} \quad \text{and} \quad n_y = \frac{\frac{\partial s}{\partial y}}{|\nabla s|} .$$

The unit vector tangent to s is any vector of unit length which is a solution to $\vec{n} \cdot \vec{m} = 0$. In order that $\vec{n}$ and $\vec{m}$ form a right handed coordinate system choose

$$\vec{m} = -n_y \vec{i} + n_x \vec{j} . \tag{2.5}$$

At a material boundary the normal component of the velocity vanishes.

The velocity $\vec{w}$ can be expressed

$$\vec{w} = (\vec{w} \cdot \vec{n})\vec{n} + (\vec{w} \cdot \vec{m})\vec{m} .$$

Thus, the velocity at a material boundary satisfies

$$\vec{w} \cdot \vec{n} = 0, \tag{2.6}$$

At a free surface the normal and tangential components of the stress must vanish.

The stress $\vec{\sigma}$ at a point on a free surface with normal $\vec{n}$ is

$$\vec{\sigma} = \Pi \cdot \vec{n} .$$

Here Π is the stress tensor

$$\Pi = \begin{bmatrix} \tau_{xx}\vec{i}\,\vec{i} & \tau_{xy}\vec{i}\,\vec{j} \\ \\ \tau_{yx}\vec{j}\,\vec{i} & \tau_{yy}\vec{j}\,\vec{j} \end{bmatrix},$$

Therefore, $\vec{\sigma}$ is given by

$$\vec{\sigma} = \begin{bmatrix} \tau_{xx}\vec{i}\,\vec{i} & \tau_{xy}\vec{i}\,\vec{j} \\ \tau_{yx}\vec{j}\,\vec{i} & \tau_{yy}\vec{j}\,\vec{j} \end{bmatrix} \begin{bmatrix} n_x\vec{i} \\ n_y\vec{j} \end{bmatrix}$$

$$= (n_x\tau_{xx} + n_y\tau_{xy})\vec{i} + (n_x\tau_{yx} + n_y\tau_{yy})\vec{j} .$$

Since Equations (2. 4) and (2. 5) can be solved for $\vec{i}$ and $\vec{j}$ to yield

$$\vec{i} = n_x\vec{n} - n_y\vec{m} ,$$

$$\vec{j} = n_y\vec{n} + n_x\vec{m} .$$

$\vec{\sigma}$ can be expressed as

$$\vec{\sigma} = (n_x\tau_{xx}+n_y\tau_{xy})(n_x\vec{n}-n_y\vec{m}) + (n_x\tau_{yx}+n_y\tau_{yy})(n_y\vec{n}+n_x\vec{m})$$

setting

$$\vec{\sigma} = \sigma_n\vec{n} + \sigma_m\vec{m}$$

$$\sigma_n = (n_x^2\tau_{xx}+n_xn_y\tau_{xy}+n_xn_y\tau_{yx}+n_y^2\tau_{yy})$$

$$\sigma_m = (-n_xn_y\tau_{xx}-n_y^2\tau_{xy}+n_x^2\tau_{yx}+n_xn_y\tau_{yy}).$$

In general for a Newtonian fluid

$$\Pi = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix} + \mu \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} + \mu \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{bmatrix} - \frac{2}{3}\mu \begin{bmatrix} \nabla\cdot\vec{w} & 0 \\ 0 & \nabla\cdot\vec{w} \end{bmatrix}$$

For an incompressible fluid the last bracketed term vanishes.

Substituting the Π components into the equation for σ_n,

$$\sigma_n = n_x^2(-P+2\mu\frac{\partial u}{\partial x}) + 2n_x n_y \mu(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}) + n_y^2(-P+2\mu\frac{\partial v}{\partial y}).$$

Using the condition that $n_x^2 + n_y^2 = 1$, this may be rewritten as

$$\sigma_n = -P + 2n_x^2\mu\frac{\partial u}{\partial x} + 2n_x n_y \mu(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}) + 2n_y^2\mu\frac{\partial v}{\partial y}.$$

Similarly, if the components of Π are substituted into the equation for σ_m the result is

$$\sigma_m = 2n_x n_y \mu(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}) + (n_x^2-n_y^2)\mu(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}) .$$

The free surface boundary condition is expressed by setting σ_n and σ_m equal to zero. Thus,

$$P = 2n_x^2\mu\frac{\partial u}{\partial x} + 2n_x n_y \mu(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}) + 2n_y^2\mu\frac{\partial v}{\partial y}; \qquad (2.7)$$

$$2n_x n_y(\frac{\partial v}{\partial y}-\frac{\partial u}{\partial x}) + (n_x^2-n_y^2)(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}) = 0. \qquad (2.8)$$

The density at an outlet must satisfy

$$\frac{\partial \rho}{\partial n} = 0.$$

This condition is also frequently used at an inlet but is not necessary in all cases.

Two inlet velocity boundary conditions have been successfully used. One holds the inlet velocity constant; that is, the other requires the normal derivative to vanish at the inlet.

$$(\vec{n} \cdot \nabla)\vec{w} = 0.$$

Any initial condition may be assigned for ρ, $\vec{w}$ or P.

III. DIFFERENCE EQUATIONS

The general method of solution of the system of partial differential equations (2. 1)-(2. 3) will be to represent the continuous variables x, y, and t as multiples of δx, δy, and δt. Then the partial differential equations can be approximated by finite difference equations and solved numerically for $\vec{w}$, ρ, and P, at $x = i\delta x$, $y = j\delta y$, and $t = n\delta t$ for discrete index values of i, j, and n.

The choice of the difference operator and the choice of the values of i, j, and n for which to define the variables are different aspects of the same problem: to find the best approximations to Equations (2. 1)-(2. 3).

Variable Placement

The region in which the flow takes place is covered by a double grid system (see Figure 1). The solid grid divides the system into cells; the dashed grid is used for variable placement. The horizontal component of velocity is defined at the sides of a cell, the vertical at the top and bottom. Pressure, density and viscosity are defined at the center. Although there are placements of the field variables relative to the mesh difference from that shown in Figure 1, Harlow (Welch *et al.*, 1966) reports that this is the only one currently developed which satisfies the physical laws.

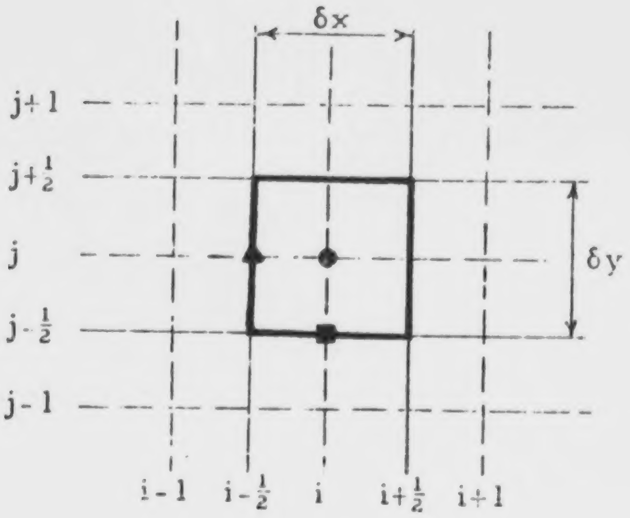


Figure 1. The double Eulerian mesh. The locations of the fluid variables are indicated by, ●: ρ, P; ▲: u; ■: v. The ijth cell is highlighted.

Before the Navier-Stokes equation is finite differenced it is convenient to put it into a slightly different form. Substituting Equations (2.1) and (2.2) into the left side of Equation (2.3) and simplifying, the Navier-Stokes equation, written separately in the $\vec{i}$ and $\vec{j}$ directions becomes

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) = \rho g_x - \frac{\partial P}{\partial x} + 2[\frac{\partial}{\partial x}(\mu\frac{\partial u}{\partial x}) + \frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})]$$

$$+ \frac{\partial}{\partial y}[\mu(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})], \tag{3.1}$$

and

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial}{\partial x}(\rho uv) + \frac{\partial}{\partial y}(\rho v^2) = \rho g_y - \frac{\partial P}{\partial y} + 2[\frac{\partial}{\partial x}(\mu\frac{\partial v}{\partial x}) + \frac{\partial}{\partial y}(\mu\frac{\partial v}{\partial y})]$$

$$\frac{\partial}{\partial x}[\mu(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})]. \tag{3.2}$$

Although the partial differential equations are equivalent, it is clear that the finite difference forms of (2.3) and (3.1), (3.2) are not. Harlow (Welch et al., 1966) has shown that the finite difference analogy of (3.1) and (3.2) satisfied Newton's Second Law more precisely than the analogous form of (2.3). The system of Equations (2.1), (2.2), (3.1) and (3.2) can be written in finite difference form as follows:

$$\frac{\rho_{ij}^{n+1} - \rho_{ij}}{\delta t} + \frac{(\rho u)_{i+\frac{1}{2}j} - (\rho u)_{i-\frac{1}{2}j}}{\delta x} + \frac{(\rho v)_{ij+\frac{1}{2}} - (\rho v)_{ij-\frac{1}{2}}}{\delta y} = 0. \tag{3.3}$$

$$\frac{u_{i+\frac{1}{2}j} - u_{i-\frac{1}{2}j}}{\delta x} + \frac{v_{ij+\frac{1}{2}} - v_{ij-\frac{1}{2}}}{\delta y} = 0. \tag{3.4}$$

$$(\rho u)_{i+\frac{1}{2}j}^{n+1} = \xi_{i+\frac{1}{2}j} + \frac{\delta t}{\delta x}(P_{ij} - P_{i+1j}), \tag{3.5}$$

where

$$\xi_{i+\frac{1}{2}j} = (\rho u)_{i+\frac{1}{2}j} + \delta t \quad \frac{(\rho u^2)_{ij} - (\rho u^2)_{i+1j}}{\delta x} + \frac{(\rho uv)_{i+\frac{1}{2}j-\frac{1}{2}} - (\rho uv)_{i+\frac{1}{2}j+\frac{1}{2}}}{\delta y}$$

$$+ \frac{2}{(\delta x)^2}[\mu_{i+1j}(u_{i+\frac{3}{2}j} - u_{i+\frac{1}{2}j}) - \mu_{ij}(u_{i+\frac{1}{2}j} - u_{i-\frac{1}{2}j})]$$

$$+ \frac{1}{\delta y} \mu_{i+\frac{1}{2}j+\frac{1}{2}} \quad \frac{u_{i+\frac{1}{2}j+1} - u_{i+\frac{1}{2}j}}{\delta y} + \frac{v_{i+1j+\frac{1}{2}} - v_{ij+\frac{1}{2}}}{\delta x}$$

$$- \mu_{i+\frac{1}{2}j-\frac{1}{2}} \quad \frac{u_{i+\frac{1}{2}j} - u_{i+\frac{1}{2}j-1}}{\delta y} + \frac{v_{i+1j-\frac{1}{2}} - v_{ij-\frac{1}{2}}}{\delta x} \qquad + \rho_{i+\frac{1}{2}j} g_x ,$$

$$(\rho v)_{ij+\frac{1}{2}}^{n+1} = \zeta_{ij+\frac{1}{2}} + \frac{\delta t}{\delta y}(P_{ij} - P_{ij+1}), \tag{3.6}$$

where

$$\zeta_{ij+\frac{1}{2}} = (\rho v)_{ij+\frac{1}{2}} + \delta t \quad \frac{(\rho uv)_{i-\frac{1}{2}j+\frac{1}{2}} - (\rho uv)_{i+\frac{1}{2}j+\frac{1}{2}}}{\delta x} + \frac{(\rho v^2)_{ij} - (\rho v^2)_{ij+1}}{\delta y}$$

$$+ \frac{2}{(\delta y)^2}[\mu_{ij+1}(v_{ij+\frac{3}{2}} - v_{ij+\frac{1}{2}}) - \mu_{ij}(v_{ij+\frac{1}{2}} - v_{ij-\frac{1}{2}})]$$

$$+ \frac{1}{\delta x} \quad \mu_{i+\frac{1}{2}j+\frac{1}{2}} \quad \frac{u_{i+\frac{1}{2}j+1} - u_{i+\frac{1}{2}j}}{\delta y} + \frac{v_{i+1j+\frac{1}{2}} - v_{ij+\frac{1}{2}}}{\delta x}$$

$$- \mu_{i-\frac{1}{2}j+\frac{1}{2}} \quad \frac{u_{i-\frac{1}{2}j+1} - u_{i-\frac{1}{2}j}}{\delta y} + \frac{v_{ij+\frac{1}{2}} - v_{i-1j+\frac{1}{2}}}{\delta x} \quad + \rho_{ij+\frac{1}{2}} g_y \, .$$

Equations(3. 3)-(3. 6) require quantities which have not yet been defined. For terms involving variables where they have not been defined, e. g., $\rho_{i+\frac{1}{2}j}$, an average of defined quantities is used. Thus

$$\rho_{i+\frac{1}{2}j} = \frac{1}{2}(\rho_{i+1j} + \rho_{ij})$$

For terms involving products such as $(\rho u)_{i+\frac{1}{2}j}$ a product of the respective quantities is used.

$$(\rho u)_{i+\frac{1}{2}j} = (\rho_{i+\frac{1}{2}j})(u_{i+\frac{1}{2}j})$$

$$= \frac{1}{2}(\rho_{i+1j} + \rho_{ij})(u_{i+\frac{1}{2}j}).$$

The only exception is the momentum flux terms such as $(\rho uv)_{i+\frac{1}{2}j-\frac{1}{2}}$ which are evaluated

$$(\rho uv)_{i+\frac{1}{2}j-\frac{1}{2}} = \begin{cases} \left(\dfrac{\rho_{ij-1}+\rho_{i+1j-1}}{2}\right)(u_{i+\frac{1}{2}j-1})\left(\dfrac{v_{ij-\frac{1}{2}}+v_{i+1j-\frac{1}{2}}}{2}\right) & \\ \qquad\qquad \text{if} \quad \left(\dfrac{v_{ij-\frac{1}{2}}+v_{i+1j-\frac{1}{2}}}{2}\right) \geq 0 & \\ \left(\dfrac{\rho_{ij}+\rho_{i+1j}}{2}\right)(u_{i+\frac{1}{2}j})\left(\dfrac{v_{ij-\frac{1}{2}}+v_{i+1j-\frac{1}{2}}}{2}\right) & \\ \qquad\qquad \text{if} \quad \left(\dfrac{v_{ij-\frac{1}{2}}+v_{i+1j-\frac{1}{2}}}{2}\right) < 0 & \end{cases}$$

A similar prescription applies to the other momentum flux terms.

For computational purposes it is convenient to put Equations (3. 3)-(3. 6) in a different form. Equation (3. 3) becomes

$$\rho^{n+1}_{ij} = \rho_{ij} - \delta t \; \frac{(\rho u)_{i+\frac{1}{2}j}-(\rho u)_{i-\frac{1}{2}j}}{\delta x} + \frac{(\rho v)_{ij+\frac{1}{2}}-(\rho v)_{ij-\frac{1}{2}}}{\delta y} \qquad (3.\ 7)$$

Equation (3. 4) can be solved for $u^{n+1}_{i-\frac{1}{2}j}$;

$$u^{n+1}_{i+\frac{1}{2}j} = \frac{\xi_{i+\frac{1}{2}j}}{\rho^{n+1}_{i+\frac{1}{2}j}} + \frac{\delta t}{\delta x}\,\frac{(P_{ij}-P_{i+1j})}{\rho^{n+1}_{i+\frac{1}{2}j}} \,. \qquad (3.8)$$

Equation (3. 8) can be written for $u^{n+1}_{i-\frac{1}{2}j}$ as

$$u^{n+1}_{i-\frac{1}{2}j} = \frac{\xi_{i-\frac{1}{2}j}}{\rho^{n+1}_{i-\frac{1}{2}j}} + \frac{\delta t}{\delta x}\,\frac{(P_{i-1j}-P_{ij})}{\rho^{n+1}_{i-\frac{1}{2}j}} \,. \qquad (3.\ 9)$$

For the v components

$$v^{n+1}_{ij+\frac{1}{2}} = \frac{\zeta_{ij+\frac{1}{2}}}{\rho^{n+1}_{ij+\frac{1}{2}}} + \frac{\delta t}{\delta y}\,\frac{(P_{ij}-P_{ij+1})}{\rho^{n+1}_{ij+\frac{1}{2}}} \;, \qquad (3.\ 10)$$

$$v^{n+1}_{ij-\frac{1}{2}} = \frac{\zeta_{ij-\frac{1}{2}}}{\rho^{n+1}_{ij-\frac{1}{2}}} + \frac{\delta t}{\delta y}\,\frac{(P_{ij-1} - P_{ij})}{\rho^{n+1}_{ij-\frac{1}{2}}} \,. \qquad (3.\,11)$$

If Equations (3. 7)-(3. 10) are substituted into the continuity equation (3. 3) for $t = (n+1)\delta t$, the result is

$$\frac{1}{\delta x}\;\frac{\xi_{i+\frac{1}{2}j}}{\rho^{n+1}_{i+\frac{1}{2}j}} - \frac{\xi_{i-\frac{1}{2}j}}{\rho^{n+1}_{i-\frac{1}{2}j}} + \frac{\delta t}{\delta x}\;\frac{P_{ij} - P_{i+1j}}{\rho^{n+1}_{i+\frac{1}{2}j}} - \frac{P_{i-1j} - P_{ij}}{\rho^{n+1}_{i-\frac{1}{2}j}}$$

$$+ \frac{1}{\delta y}\;\frac{\zeta_{ij+\frac{1}{2}}}{\rho^{n+1}_{ij+\frac{1}{2}}} - \frac{\zeta_{ij-\frac{1}{2}}}{\rho^{n+1}_{ij-\frac{1}{2}}} + \frac{\delta t}{\delta x}\;\frac{P_{ij} - P_{ij+1}}{\rho^{n+1}_{i+\frac{1}{2}j}} - \frac{P_{ij-1} - P_{ij}}{\rho^{n+1}_{ij-\frac{1}{2}}} = 0.$$

This may be put in the form

$$P_{ij} = B^{1}_{ij}P_{i+1j} + B^{2}_{ij}P_{i-1j} + B^{3}_{ij}P_{ij+1} + B^{4}_{ij}P_{ij-1} + A_{ij}. \qquad (3.\,12)$$

The coefficients are given by

$$A_{ij} = \frac{1}{C_{ij}}\;\frac{1}{\delta x}\;\frac{\xi_{i+\frac{1}{2}j}}{\rho^{n+1}_{i+\frac{1}{2}j}} - \frac{\xi_{i-\frac{1}{2}j}}{\rho^{n+1}_{i-\frac{1}{2}j}} + \frac{1}{\delta y}\;\frac{\zeta_{ij+\frac{1}{2}}}{\rho^{n+1}_{ij+\frac{1}{2}}} - \frac{\zeta_{ij-\frac{1}{2}}}{\rho^{n+1}_{ij-\frac{1}{2}}} \,,$$

$$B^{1}_{ij} = \frac{1}{C_{ij}}\,\frac{\delta t}{\delta x^{2}}\,\frac{1}{\rho^{n+1}_{i+\frac{1}{2}j}} \,,$$

$$B^{2}_{ij} = \frac{1}{C_{ij}}\,\frac{\delta t}{\delta x^{2}}\,\frac{1}{\rho^{n+1}_{i-\frac{1}{2}j}} \,,$$

$$B^3_{ij} = \frac{1}{C_{ij}} \frac{\delta t}{\delta y^2} \frac{1}{\rho^{n+1}_{ij+\frac{1}{2}}} ,$$

$$B^4_{ij} = \frac{1}{C_{ij}} \frac{\delta t}{\delta y^2} \frac{1}{\rho^{n+1}_{ij-\frac{1}{2}}} ,$$

and

$$C_{ij} = \frac{\delta t}{\delta x^2} \quad \frac{1}{\rho^{n+1}_{i+\frac{1}{2}j}} + \frac{1}{\rho^{n+1}_{i-\frac{1}{2}j}} \quad + \frac{\delta t}{\delta y^2} \quad \frac{1}{\rho^{n+1}_{ij+\frac{1}{2}}} + \frac{1}{\rho^{n+1}_{ij-\frac{1}{2}}} \quad .$$

Differenced Boundary Conditions

The region in which the fluid motion occurs has been covered with a mesh. It is necessary to approximate the boundary of the fluid, s, in terms of line segments for the mesh. The algorithm requires quantities from surrounding cells for the calculations in any particular cell. Thus to calculate quantities near a boundary, it is necessary to create a layer of image cells outside the boundary of the fluid. The quantities for these cells are determined by the boundary conditions at the interface of the image and actual cells. In this way the boundary conditions are accounted for in the algorithm.

After the boundary has been "rectangularized" into line segments of the mesh, all cells are flagged according to the following scheme.

I. Interior cells

   A. EMP = cell containing no fluid particles.

   B. SUR = cell containing particles adjacent to an EMP cell.

   C. FULL = cell containing particles with no adjacent EMP cell.

   D. REG = interior cell containing particles.

II. Boundary (BND) cells.

- A. OUT = cell defining outlet.
- B. Inlet cells.
    1. INC = inlet cell with constant velocity.
    2. INM = inlet cell with velocity matching the adjacent interior cell.
- C. Rigid boundaries.
    1. NOSLP = boundary cell with no tangential component of velocity.
    2. FRSLP = boundary cell with tangential component of velocity equal to adjacent interior cell.
    3. COR = boundary cell with interior cells on two sides, may be either FRSLP or NOSLP.
- D. EMPBDN = BND cell that is needed only for indexing purposes.

Boundary cells never change flags; interior cells may change flags as particles enter or vacate a cell. Figure 2 shows how the cells are flagged for a typical problem.

Figure 3 depicts a boundary between a cell and its image. The quantities $u_{i-\frac{3}{2}j}$, $u_{i-\frac{1}{2}j}$, $v_{i-1j-\frac{1}{2}}$, ρ_{i-1j}, and P_{i-1j} are needed in the calculations and must be determined from the boundary conditions. All types of boundary conditions are derived for a cell and a boundary oriented as in Figure 3. All other orientations of boundaries are analogous. For a boundary oriented as in Figure 3 $n_x = 1$ and $n_y = 0$.

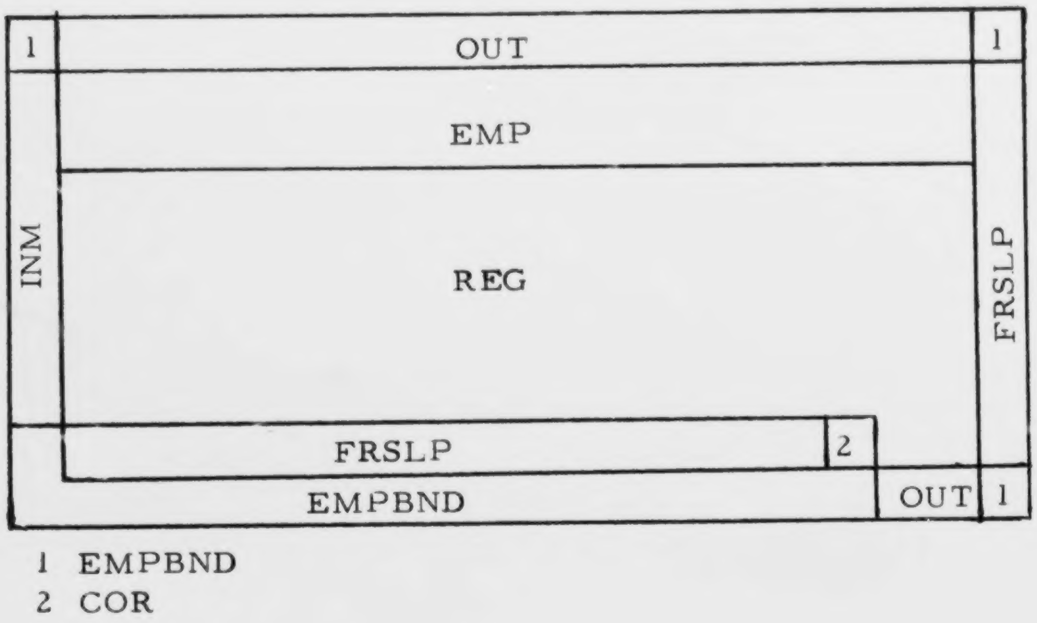

1
OUT
1
EMP
INM
REG
FRSLP
FRSLP
2
EMPBND
OUT
1
1 EMPBND
2 COR

Figure 2. Cell flags for a typical reservoir problem.

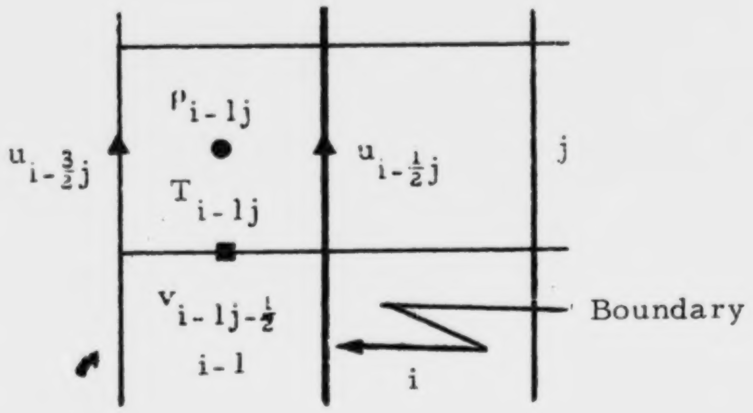

ρ_{i-1j}
$u_{i-\frac{3}{2}j}$
T_{i-1j}
$u_{i-\frac{1}{2}j}$
j
$v_{i-1j-\frac{1}{2}}$
Boundary
i-1
i

Figure 3. Cell i-1, j is a boundary cell.

For all types of boundary cells,

$$\rho_{i-1j} = \rho_{ij} .$$

At a solid wall two ways were used to determine the tangential component (in this case $v_{i-1j-\frac{1}{2}}$) of velocity. A wall was NOSLP if

$$\vec{w} = 0 \, .$$

A wall was FRSLP if

$$(\vec{n} \cdot \nabla) \vec{w} = 0 .$$

Continuity in the boundary cell was used to calculate $u_{i-\frac{3}{2}j}$. The pressure boundary value was found by substituting the density and velocity values into Equation (3.5) written for the $i-1j^{th}$ cell.

FRSLP Boundary Conditions:

$$u_{i-\frac{1}{2}j} = 0$$

$$u_{i-\frac{3}{2}j} = -u_{i+\frac{1}{2}j}$$

$$v_{i-1j-\frac{1}{2}} = v_{ij-\frac{1}{2}}$$

$$P_{i-1j} = P_{ij} - \rho_{ij} g_x \delta x$$

NOSLP Boundary Conditions:

$$u_{i-\frac{1}{2}j} = 0$$

$$u_{i-\frac{3}{2}j} = u_{i+\frac{1}{2}j}$$

$$v_{i-1j-\frac{1}{2}} = -v_{ij-\frac{1}{2}}$$

$$P_{i-1j} = P_{ij} - \frac{4\mu_{ij}u_{i+\frac{1}{2}j}}{\delta x} - \frac{v_{ij+\frac{1}{2}}(\mu_{ij}+\mu_{ij+1}) - v_{ij-\frac{1}{2}}(\mu_{ij}+\mu_{ij-1})}{\delta y}$$

$$- \rho_{ij}g_x\delta x$$

The boundary condition at an inlet is also of two different types. If the velocity was constant and normal to the boundary,

$$u_{i-\frac{1}{2}j} = u_{IN}$$

$$v_{i-1j-\frac{1}{2}} = v_{ij-\frac{1}{2}}$$

If the velocity profile matched the profile inside the boundary,

$$u_{i-\frac{3}{2}j} = u_{i-\frac{1}{2}j} = u_{i+\frac{1}{2}j}$$

$$v_{i-1j-\frac{1}{2}} = v_{ij-\frac{1}{2}}$$

As for the solid wall, the pressure is found from Equation (3. 5).

INC Boundary Conditions:

$$u_{i-\frac{1}{2}j} = u_{IN}$$

$$v_{i-1j-\frac{1}{2}} = -v_{ij-\frac{1}{2}}$$

$$u_{i-\frac{3}{2}j} = u_{i+\frac{1}{2}j}$$

$$P_{i-1j} = P_{ij} - \rho_{ij}g_x\delta x - \frac{2\mu_{ij}}{\delta x}(u_{i+\frac{1}{2}j} - u_{IN})$$

INM Boundary Conditions:

$$u_{i-\frac{3}{2}j} = u_{i-\frac{1}{2}j} = u_{i+\frac{1}{2}j}$$

$$v_{i-1j-\frac{1}{2}} = v_{ij-\frac{1}{2}}$$

$$P_{i-1j} = P_{ij}$$

At an outlet $u_{i-\frac{1}{2}j}$ and $u_{i-\frac{3}{2}j}$ were calculated from continuity. A satisfactory pressure boundary condition was found to be

$$P_{i-1j} = P_{ij} \ .$$

It was assumed that the fluid did not accelerate as it left through an OUT cell.[1] Thus

$$v_{i-1j-\frac{1}{2}} = v_{ij-\frac{1}{2}} \ .$$

OUT Boundary Conditions:

$$u_{i-\frac{1}{2}j} = u_{i+\frac{1}{2}j} + \frac{\delta x}{\delta y}(v_{ij+\frac{1}{2}} - v_{ij-\frac{1}{2}})$$

$$v_{i-1j-\frac{1}{2}} = v_{ij-\frac{1}{2}}$$

$$u_{i-\frac{3}{2}j} = u_{i-\frac{1}{2}j} + \frac{\delta x}{\delta y}(v_{i-1j+\frac{1}{2}} - v_{i-1j-\frac{1}{2}})$$

$$P_{i-1j} = P_{ij}$$

---

[1] With acceleration terms from Equation (3.5) present, velocities at the outlets were much too large.

At a free surface, Equation (2. 7) may be differenced for a pressure boundary value. As pointed out by Hirt and Shannon (1969) for the free surface velocity boundary condition, Equation (2. 8) is difficult to apply without knowing the exact location of the free surface. The approximation for the free surface in terms of the differencing grid is inadequate; there seems to be no general method for determining the free surface in terms of the fluid particles. Continuity does not yield a surface velocity boundary condition since it is valid only for regions completely filled with fluid. An improved approach due to Chan (1969) involves interpolation for the velocity boundary values from "within" the fluid toward the free surface. The cases when there are one, two, three, or four EMP cells surrounding a SUR cell are considered.

With one empty cell as in Figure 4a the velocity $v_{ij+\frac{1}{2}}$ is interpolated[2] and the pressure is calculated from Equation (2. 7)

$$v_{ij+\frac{1}{2}} = 2v_{ij-\frac{1}{2}} - v_{ij-\frac{3}{2}}$$

$$P_{ij} = \frac{2\mu_{ij}}{\delta y} (v_{ij+\frac{1}{2}} - v_{ij-\frac{1}{2}})$$

With two empty cells as in Figure 4b both velocities are interplated.[2] The pressure is calculated from Equation (2. 7) using

---

[2] When it is impossible to interpolate because the cell across from the free surface is a BND cell, the free surface value is set equal to the boundary value.

$n_x = \frac{1}{\sqrt{2}}$ and $n_y = \frac{1}{\sqrt{2}}$. Thus

$$u_{i+\frac{1}{2}j} = 2u_{i+\frac{1}{2}j-1} - u_{i+\frac{1}{2}j-2},$$

$$v_{ij+\frac{1}{2}} = 2v_{ij-\frac{1}{2}} - v_{ij-\frac{3}{2}},$$

$$P_{ij} = \frac{\mu_{ij}}{2} \frac{1}{\delta y} (u_{i-\frac{1}{2}j} + u_{i+\frac{1}{2}j} - u_{i-\frac{1}{2}j-1} - u_{i+\frac{1}{2}j-1})$$

$$+ \frac{1}{\delta x} (v_{ij+\frac{1}{2}} + v_{ij-\frac{1}{2}} - v_{i-1j+\frac{1}{2}} - v_{i-1j-\frac{1}{2}}).$$

If there are three empty cells as in Figure 4c, the value of $v_{ij+\frac{1}{2}}$ is interpolated.[3] The horizontal values $u_{i+\frac{1}{2}j}$ and $u_{i-\frac{1}{2}j}$ are calculated using the values from the previous time cycle.

$$u_{i+\frac{1}{2}j} = (u_{i+\frac{1}{2}j})_{OLD} + g_x \delta t$$

$$u_{i-\frac{1}{2}j} = u_{i+\frac{1}{2}j}$$

$$v_{ij+\frac{1}{2}} = 2v_{ij-\frac{1}{2}} - v_{ij-\frac{1}{2}}$$

$$P_{ij} = 0$$

Finally, for four empty cells as in Figure 4d

$$u_{i+\frac{1}{2}j} = (u_{i+\frac{1}{2}j})_{OLD} + g_x \delta t$$

$$u_{i-\frac{1}{2}j} = u_{i+\frac{1}{2}j}$$

---

[3] See footnote 2, page 24.

[4] Here in free surface boundary values is the only time velocity values are needed from a previous time cycle.

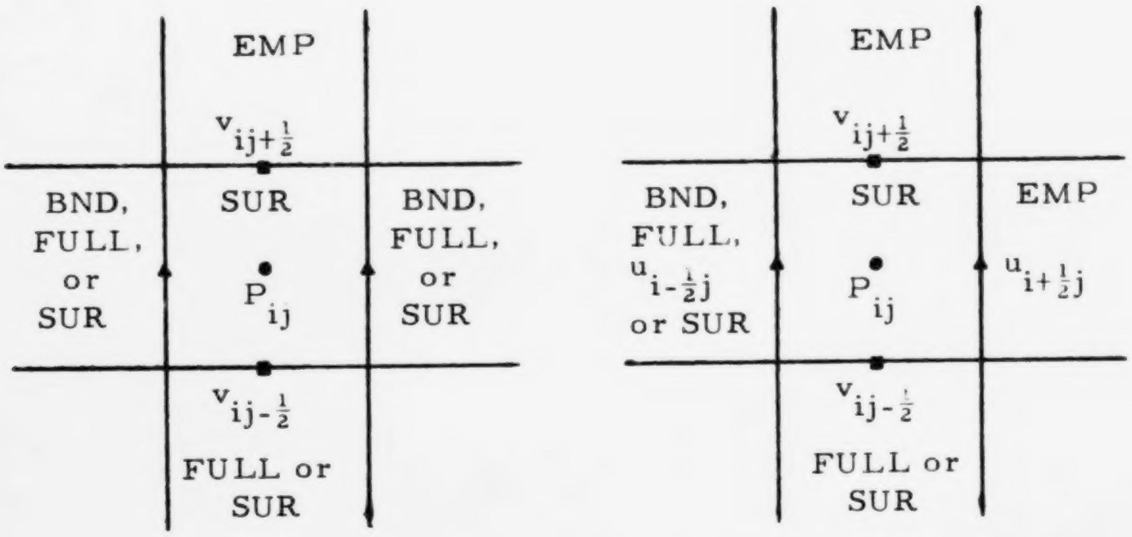

EMP
$v_{ij+\frac{1}{2}}$
BND, FULL, or SUR
SUR
P_{ij}
BND, FULL, or SUR
$v_{ij-\frac{1}{2}}$
FULL or SUR
EMP
$v_{ij+\frac{1}{2}}$
BND, FULL, $u_{i-\frac{1}{2}j}$ or SUR
SUR
P_{ij}
EMP
$u_{i+\frac{1}{2}j}$
$v_{ij-\frac{1}{2}}$
FULL or SUR

Figure 4a. One empty cell.

Figure 4b. Two empty cells.

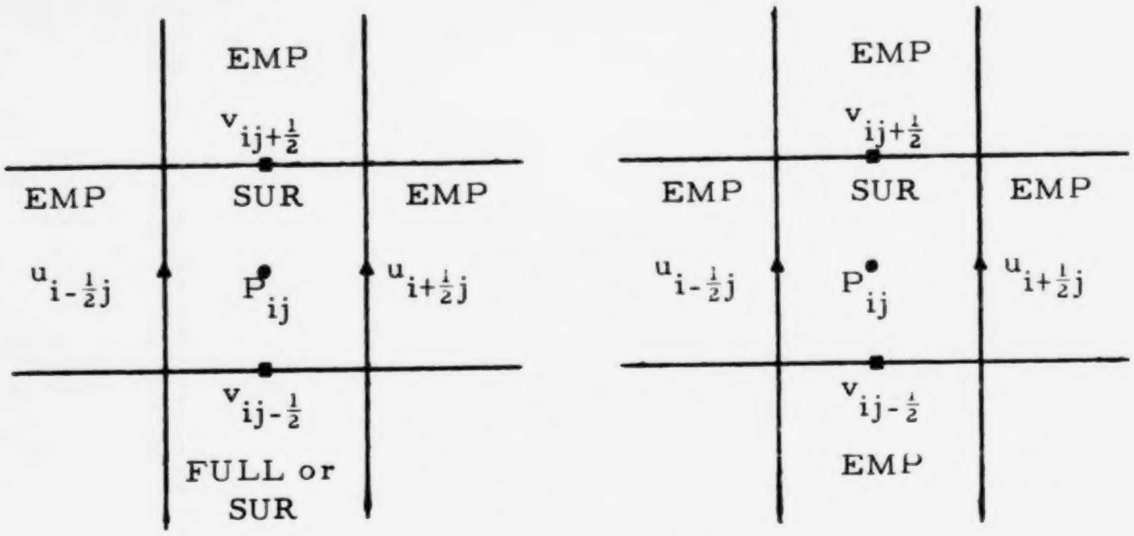

EMP
$v_{ij+\frac{1}{2}}$
EMP
$u_{i-\frac{1}{2}j}$
SUR
P_{ij}
EMP
$u_{i+\frac{1}{2}j}$
$v_{ij-\frac{1}{2}}$
FULL or SUR
EMP
$v_{ij+\frac{1}{2}}$
EMP
$u_{i-\frac{1}{2}j}$
SUR
P_{ij}
EMP
$u_{i+\frac{1}{2}j}$
$v_{ij-\frac{1}{2}}$
EMP

Figure 4c. Three empty cells.

Figure 4d. Four empty cells.

Figure 4. SUR cell configurations.

$$v_{ij+\frac{1}{2}} = (v_{ij+\frac{1}{2}})_{OLD} + g_x \delta t$$

$$v_{ij-\frac{1}{2}} = v_{ij+\frac{1}{2}}$$

$$P_{ij} = 0$$

If the boundary cell is a corner cell (see Figure 5), special calculations are required. There are two types of corner cells depending on whether the boundary in which the corner occurs is FRSLP or NOSLP. For either type

$$u_{i+\frac{1}{2}j} = 0,$$

$$v_{ij+\frac{1}{2}} = 0.$$

If the boundary is FRSLP,

$$u_{i-\frac{1}{2}j} = u_{i-\frac{1}{2}j+1},$$

$$v_{ij-\frac{1}{2}} = v_{i+1j-\frac{1}{2}}.$$

The pressure P_{ij} is different in the calculation of P_{i+1j} and P_{ij+1}. For P_{ij+1}

$$P_{ij} = P_{ij+1} - \rho_{ij+1} g_y \delta y,$$

while for P_{i+1j}

$$P_{ij} = P_{i+1j} - \rho_{i+1j} g_x \delta x.$$

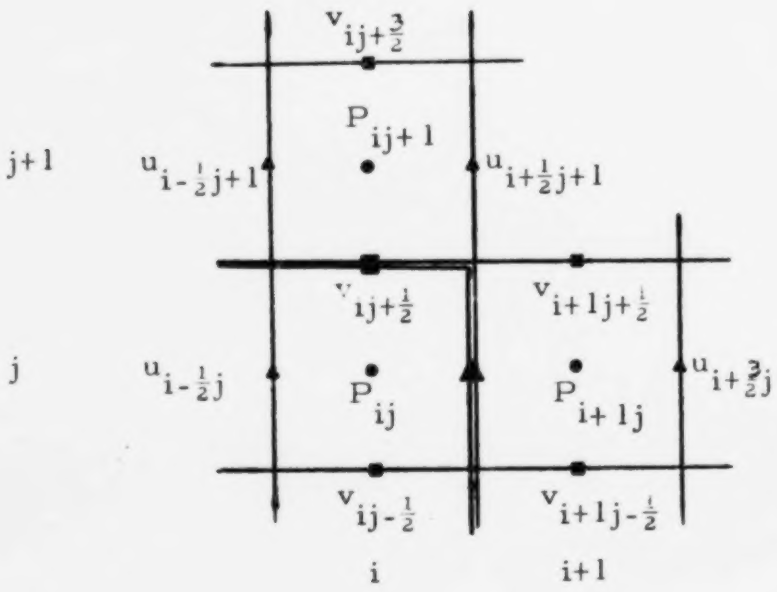

$v_{ij+\frac{3}{2}}$
P_{ij+1}
j+1
$u_{i-\frac{1}{2}j+1}$
$u_{i+\frac{1}{2}j+1}$
$v_{ij+\frac{1}{2}}$
$v_{i+1j+\frac{1}{2}}$
j
$u_{i-\frac{1}{2}j}$
P_{ij}
P_{i+1j}
$u_{i+\frac{3}{2}j}$
$v_{ij-\frac{1}{2}}$
$v_{i+1j-\frac{1}{2}}$
i
i+1

Figure 5. COR cell.

If the boundary is NOSLP,

$$u_{i-\frac{1}{2}j} = -u_{i-\frac{1}{2}j+1},$$

$$v_{ij-\frac{1}{2}} = -v_{i+1j-\frac{1}{2}}.$$

The value of P_{ij} for the calculation of P_{ij+1} is found by substituting the above velocity values into Equation (3.6). Similarly, find P_{ij} for P_{i+1j} from Equation (3.5).

In calculation of $\xi_{i+\frac{1}{2}j+1}$ only:

for FRSLP, $u_{i+\frac{1}{2}j} = u_{i+\frac{1}{2}j+1}$

for NOSLP, $u_{i+\frac{1}{2}j} = -u_{i+\frac{1}{2}j+1}$

In calculation of $\zeta_{i+1j+\frac{1}{2}}$ only:

for FRSLP, $v_{ij+\frac{1}{2}} = v_{i+1j+\frac{1}{2}}$

for NOSLP, $v_{ij+\frac{1}{2}} = -v_{i+1j+\frac{1}{2}}$

In calculation of A_{i+1j} only:

$$\zeta_{i+1j+\frac{1}{2}} = \zeta_{i+1j+\frac{1}{2}} + \frac{\delta t}{4\delta x\delta y}[(2\mu_{i+1j}+\mu_{ij+1}+\mu_{i+1j+1})u_{i+\frac{1}{2}j+1}]$$

$\xi_{i+\frac{1}{2}j} = P_{i+1j} - P_{ij}$ (not literally)

In calculation of A_{ij+1} only:

$$\xi_{i+\frac{1}{2}j+1} = \xi_{i+\frac{1}{2}j+1} + \frac{\delta t}{4\delta x\delta y}[(2\mu_{ij+1}+\mu_{i+1j}+\mu_{i+1j+1})v_{i+1j+\frac{1}{2}}]$$

$\zeta_{ij+\frac{1}{2}} = P_{ij+1} - P_{ij}$ (not literally)

In calculation of horizontal velocities of particles in upper half of cell $(i+1, j)$, $u_{i+\frac{1}{2}j+1} = 0$.

In the calculation of vertical velocities of particles in right half of cell $(i, j+1)$, $v_{i+1j+\frac{1}{2}} = 0$.

IV. OVERRELAXATION

The form of Equation (3. 12) is familiar to all those who have studied numerical solution of partial differential equations. It is the usual finite difference form for solving Poisson's equation

$$\nabla^2 P(x, y) = f(x, y) \tag{4. 1}$$

Let s be a region in the x, y plane. Let δx and δy be the spacing in the x and y directions, respectively, of a grid which covers s. (It is assumed that the boundary of s is the union of a finite number of straight lines, each of which is either horizontal or vertical, although the technique described below will work for other configurations either by changing the coordinate system or by interpolating at the boundary.) Then the partial derivatives in (4. 1) may be approximated, as in Chapter III, by

$$\frac{\partial^2 P_{ij}}{\delta x^2} \simeq \frac{P_{i+1j} - 2P_{ij} + P_{i-1j}}{\delta x^2} \tag{4. 2}$$

and

$$\frac{\partial^2 P}{\delta y^2} \simeq \frac{P_{ij+1} - 2P_{ij} + P_{ij-1}}{\delta y^2} . \tag{4. 3}$$

Using (4. 2) and (4. 3) and rearranging, Equation (4. 1) may be approximated with

$$P_{ij} = \frac{1}{2\left(\frac{1}{\delta x^2} + \frac{1}{\delta y^2}\right)} \quad \frac{P_{i+1j} + P_{i-1j}}{\delta x^2} + \frac{P_{ij+1} + P_{ij-1}}{\delta y^2} - f(x_0, y_0) \quad . \qquad (4.4)$$

There are several schemes available for solving (4.4), the easiest of which is an iteration process called simple iteration. This technique consists of the following steps:

(1) An initial guess (usually zero) is made for the function P at each point (x_i, y_j) of the mesh (except, of course, at the boundaries). Call these initial values $P_{ij}^{(0)}$.

(2) Using Equation (4.4), new values, called $P_{ij}^{(1)}$, are computed using $P_{ij}^{(0)}$ and boundary values.

(3) The difference between the new values and the old values is checked against a tolerance. If the values of $P_{ij}^{(1)}$ are too far different from those of $P_{ij}^{(0)}$, new values $P_{ij}^{(2)}$, are computed from $P_{ij}^{(1)}$ as in Step (2).

(4) Steps (2) and (3) are repeated until, for some k, the $P_{ij}^{(k)}$'s are sufficiently close to the $P_{ij}^{(k-1)}$'s. At this point the iterations are stopped, the solution is said to have converged, and the process of simple iteration is said to "work" for this problem.

A more efficient scheme, called Seidel's Method or simple relaxation, is the same as simple iteration except for one refinement. Instead of using only quantities from the previous iteration to compute new ones, simple relaxation uses new values, as soon as they have been determined, in the calculation of other new values. It is clear that the time saved by using this process will be dependent on the order in which the points are taken during an iteration. An order which takes maximum advantage of the refinement over simple iteration is called a consistent order. One such ordering is to start with the lower left-hand point, work across to the right, then left-to-right on the next higher row of points. This is continued throughout the mesh, ending with the upper right-hand point. It can be shown (Forsythe and Wasow, 1960) that if simple iteration "works" whenever a consistent order is used, then simple relaxation "works" exactly twice as fast.

An even more efficient method for solving Equation (4. 4) exists. This scheme, called overrelaxation, speeds the convergence of simple relaxation by multiplying the changes between iterations by a fixed number greater than one. The following discussion will help to clarify this.

Define the new quantity, $R_{ij}^{(k)}$, called the k^{th} residual of P_{ij} in the following way:

$$R_{ij}^{(k)} = 2\left(\frac{1}{\delta x^2}+\frac{1}{\delta y^2}\right) \quad \frac{P_{i+1j}^{(\ell)}+P_{i-1j}^{(\ell)}}{\delta x^2} + \frac{P_{ij+1}^{(\ell)}+P_{ij-1}^{(\ell)}}{\delta y^2} - f(x_0, y_0) \qquad {}^{(k)} - P_{ij}^{(k-1)}, \tag{4. 5}$$

where the superscript ℓ has the value of either k or $k-1$.

Equation (4.4) can now be written:

$$P_{ij}^{(k)} = P_{ij}^{(k-1)} + R_{ij}^{(k)} . \tag{4.6}$$

This is the equation which is used in simple relaxation. A more general equation can be written to cover all types of relaxation processes:

$$P_{ij}^{(k)} = P_{ij}^{(k-1)} + qR_{ij}^{(k)} . \tag{4.7}$$

When $q < 1$, the process is termed underrelaxation; when $q = 1$, the process is the simple relaxation already discussed; and when $q > 1$, it is called overrelaxation.

Forsythe and Wasow (1960) have also shown that when a consistent order is used, if simple relaxation "works," then overrelaxation "works." The amount of time saved by using overrelaxation will, of course, depend on the overrelaxation factor, q. Using a matrix analysis of the operations involved, a relation between the rate of convergence and the overrelaxation parameter may be obtained. This relation is depicted in Figure 6; several observations can be made from this curve.

(1) Underrelaxation is not profitable; it requires more time than any other relaxation method.

(2) An optimum overrelaxation factor, q_{opt}, exists.

(3) Although q_{opt} depends on the problem being considered, its value lies in the interval $1 < q_{out} < 2$.

(4) Approaching q_{opt} from the left, the curve has an infinite slope, while the slope is one for $q \geq q_{opt} + 0$. Thus it is better to use $q = q_{opt} + \epsilon$ than to use $q = q_{opt} - \epsilon$, for some small $\epsilon > 0$.

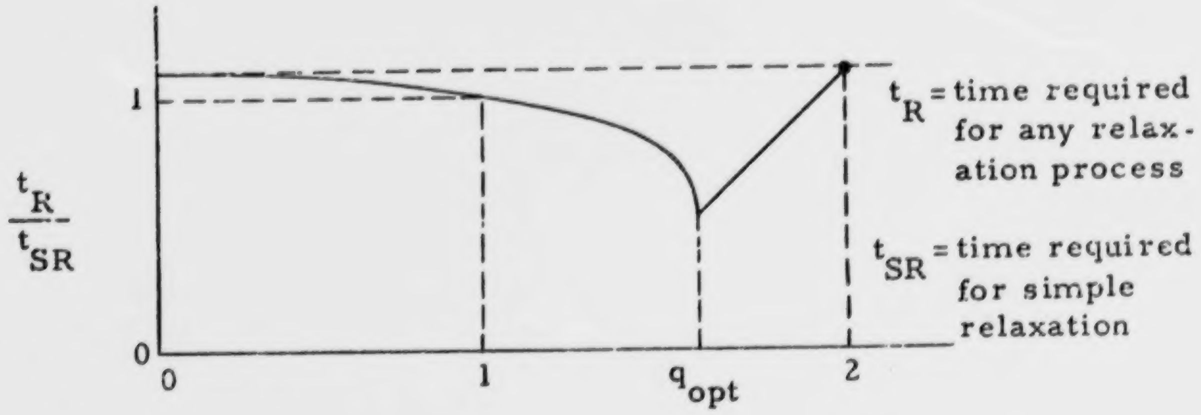


Figure 6. Overrelaxation factor curve.

In general, determination of the best overrelaxation factor to use cannot be done exactly. The next section describes a method for finding q_{opt} approximately, and also shows how to obtain q_{opt} exactly for the special case when R is a rectangle.

Determination of Overrelaxation Factor

As was noted above, the best overrelaxation factor cannot, in general, be computed exactly. However, a method, shown below, does exist for estimating q_{opt}.

If a problem is stated by using simple relaxation $(q = 1)$, then an estimate of the rate of convergence, r, will be given by

$$\frac{\|R^{(k)}\|}{\|R^{(k-1)}\|} \to r \quad \text{as} \quad k \to \infty. \tag{4.8}$$

Any matrix norm will suffice for this estimate. Fortunately, a relation exists between r and q_{opt}:

$$q_{opt} = \frac{2}{1+\sqrt{1-r}}\,. \tag{4.9}$$

Thus, one may run for, say, ten iterations using $q = 1$, form the quotient $\|R^{(10)}\| / \|R^{(9)}\|$, compute a new q from (4.9), and continue by using overrelaxation. It should be pointed out that the quotients in (4.8) will behave in a random manner when $q \neq 1$, while with $q = 1$ they will steadily decrease until r is reached.

V. THE ALGORITHM

The basic algorithm for marker-and-cell calculations can be described briefly in the following nine steps:

(1) Predict new densities using Equation (3. 7).

(2) Using these densities calculate new pressure coefficients and obtain a rough pressure by relaxing Equation (4. 7).

(3) Using these pressures calculate new velocities using Equations (3. 8) and (3. 10).

(4) Find new particle positions assuming that the particles move with this velocity field.

(5) Calculate new densities and viscosities by averaging the densities and viscosities of the particles.

(6) Compare this value of the density with the previous value. If different, go back to 2 with new densities; if same, the density values has converged. Continue.

(7) Calculate the pressures more precisely for this density.

(8) Find final velocities.

(9) Move particles

These nine steps relate all the essential features of the algorithm. Steps 1, 5, and 6 are the predictor-corrector portion. The calculation cycle continues until the density remains unchanged. Steps 2, 3, 7, and 8 are the Eulerian calculation of the variables P, u, and v. Steps 4, 5, and 9 are the Lagrangian calculation of the particle positions and the density in each cell.

In the Lagrangian calculation of the particle positions, the velocity used to move each particle is a weighted average of nearby velocities. The calculation of these weights is given below for the horizontal velocity, u.

A rectangle of dimension δx by δy is centered over the four nearest horizontal components of the velocity field. A similar rectangle is centered over the k^{th} particle. The particle rectangle and the velocity rectangles overlap (see Figure 7). Each velocity's weight is the fraction of the particle's rectangle that it covers.

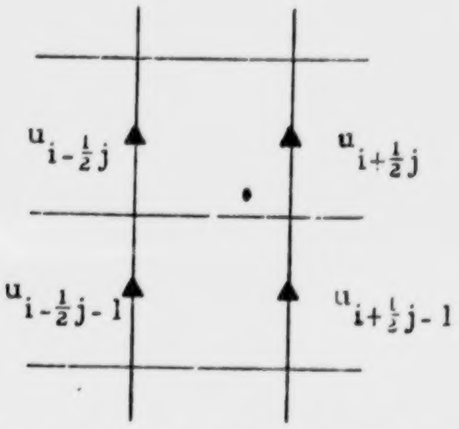


Figure 7a. A particle and the four nearest horizontal velocities.

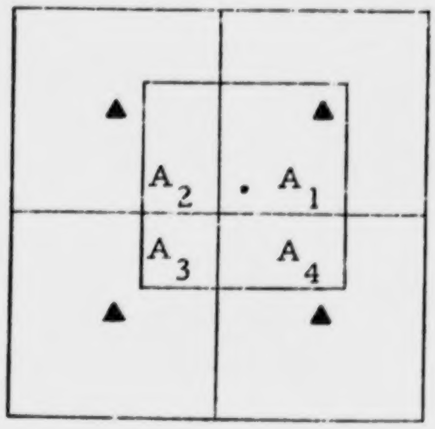


Figure 7b. The velocities and their weights.

Thus, the particle's horizontal velocity is given by

$$u_k = \frac{1}{\delta x \delta y}(A_1 u_{i+\frac{1}{2}\,j} + A_2 u_{i-\frac{1}{2}\,j} + A_3 u_{i-\frac{1}{2}\,j-1} + A_4 u_{i-\frac{1}{2}\,j-1}).$$

The particle's new x-coordinate is given by

$$x_k^{n+1} = x_k + u_k \delta t.$$

Similar calculations are performed for the vertical velocities and the y-coordinate.

The stability criteria for this procedure are reported to be (Welch, 1966):

$$C\delta t < \frac{2\delta x \delta y}{\delta x + \delta y},$$

where C is the wave speed of the fluid, and

$$2\nu\delta t < \frac{\delta x^2 \delta y^2}{\delta x^2 + \delta y^2}.$$

In addition, Shannon (1967) reports that the following criteria should also be met:

$$\delta t < \frac{\delta x^2}{4\nu},$$

$$\delta t^2 < \frac{\delta x^2}{u_{max}^2},$$

$$\delta t < \frac{\delta x}{5u_{\text{input}}} ,$$

and

$$\frac{1}{2}\,\delta t u_{\max}^2 + \frac{1}{4}\,\delta x^2 \,\frac{\partial(u_{\max})}{\delta x} < \nu .$$

Similar inequalities hold in the y direction.

Hwang (1968) derived an additional criterion for the case when the viscous and inertial forces were in relative balance

$$\delta t \leq \min \frac{2\mu h^2 u_0^2}{\rho(u_0^2+v_0^2)(h^2 u_0^2+4\nu^2)} , \frac{2\mu h^2 v_0^2}{\rho(u_0^2+v_o^2)(h^2 v_0^2+4\nu^2)} ,$$

where u_0 and v_0 are the steady-state velocity components, and h is the dimension of the square mesh cell.

The NUMAC algorithm as described above has been made into a computer program and used to examine several typical fluid flow problems. A flow chart for this program is given on the next page and a listing can be found in the Appendix.

The following is a description of the subroutines. The numbers refer to Figure 11.

1. CELSET flags the cells initially: boundary, empty boundary, full, free surface, or empty.
2. PARSET creates the initial particle configuration and assigns the particles their appropriate densities and viscosities, then

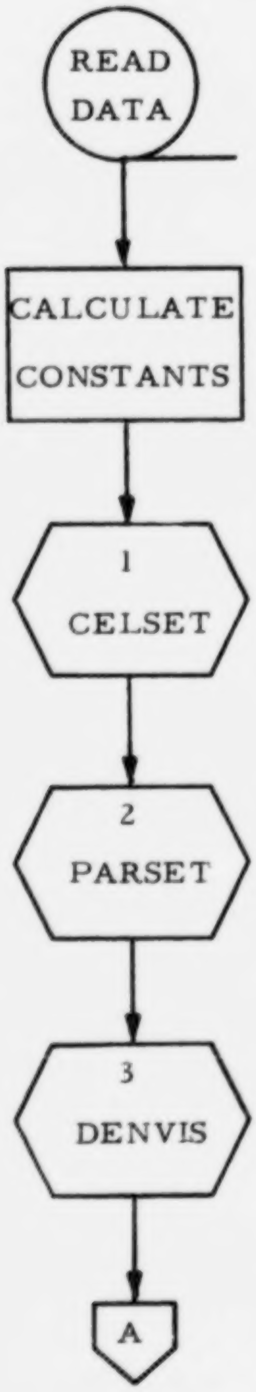

READ
DATA
CALCULATE
CONSTANTS
1
CELSET
2
PARSET
3
DENVIS
A

Figure 8. The NUMAC flow chart. Numbers refer to the list of subroutines.

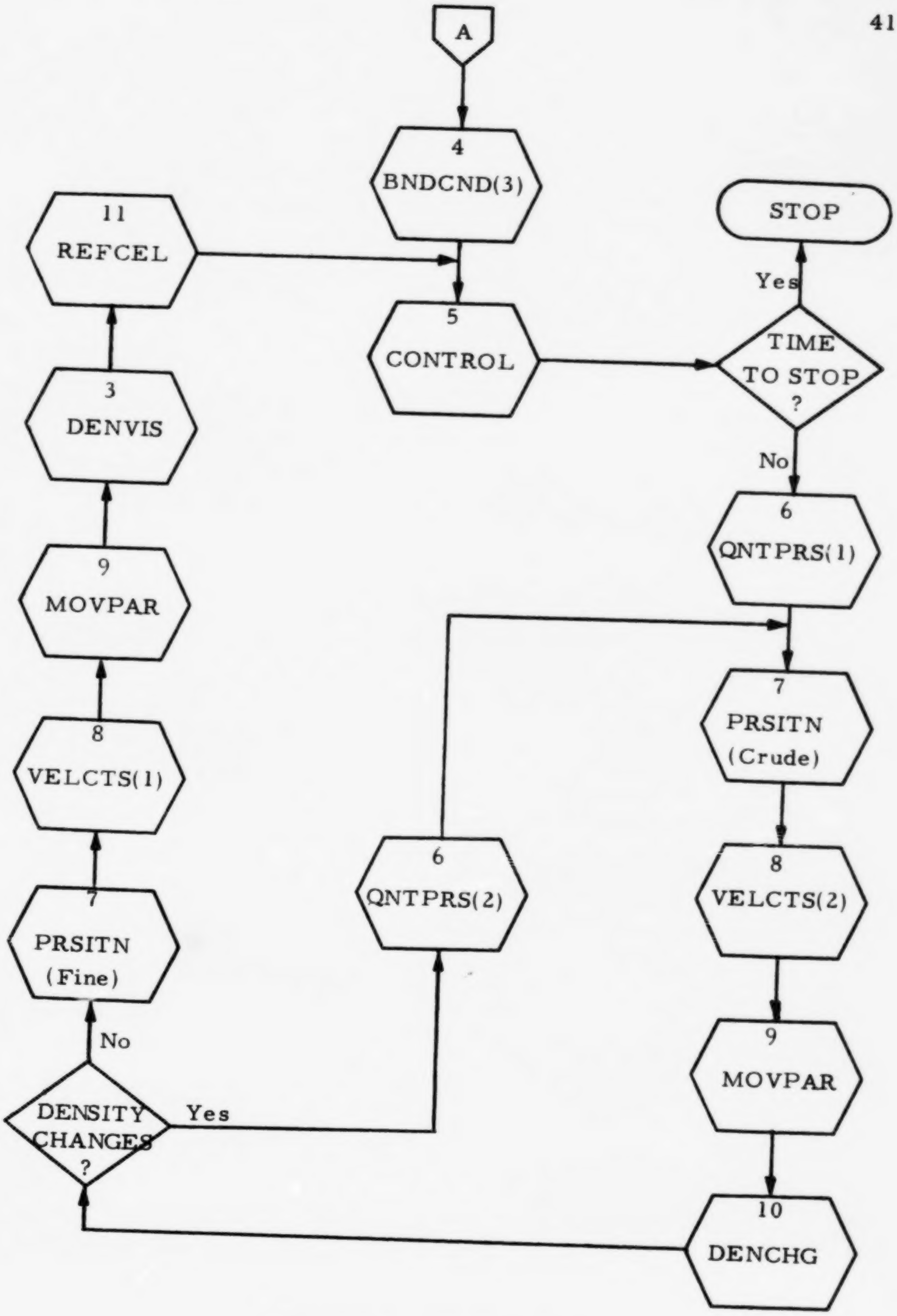

A
4
BNDCND(3)
11
REFCEL
5
CONTROL
STOP
Yes
TIME TO STOP ?
No
3
DENVIS
6
QNTPRS(1)
9
MOVPAR
7
PRSITN (Crude)
8
VELCTS(1)
6
QNTPRS(2)
8
VELCTS(2)
7
PRSITN (Fine)
No
9
MOVPAR
DENSITY CHANGES ?
Yes
10
DENCHG

Figure 8. Continued.

calls REFCEL.

3. DENVIS calculates the density and viscosity fields and boundary conditions.

4. BNDCND does the following, depending on the value of the calling argument:

   BNDCND (2) calls FSBDCD (2), calculates temporary velocity boundary conditions at solid walls, and then calls INBDCD and OTBDCD.

   BNDCND (3) calls FSBDCD (3), otherwise is the same as BNDCND (2).

   BNDCND (1) calls FSBDCD (1), and is the same as BNDCND (2) except that all the velocity boundary conditions are final.

4a. FSBDCD does the following, depending on the value of the calling argument:

   FSBDCD (1) and FSBDCD (2) calculate all the velocity boundary conditions associated with the free surface.

   FSBDCD (3) calculates only the velocities in the empty cells next to the free surface.

4b. INBDCD calculates the velocity boundary conditions at an INC or INM wall.

4c. OTBDCD calculates the velocity boundary conditions at an OUT wall.

5. CNTROL prints or plots specified information at specified intervals by calling appropriate subroutines, saves information on tape at specified intervals so the problem may be restarted later, and, depending on whether it is time to stop the calculations, either advances time (t) and continues, or stops the program.
6. QNTPRS (1) calculates ξ and ζ for each cell and B^1, B^2, B^3, B^4, and A for each full cell. QNTPRS (2) only calculates B^1, B^2, B^3, B^4, and A for certain cells: cells which change density and the four bordering cells for the two-layer model, and each full cell for the continuous density model.
7. PRSITN calculates the pressure field by using the method of overrelaxation to solve the finite-difference form of Poisson's equation. Only full cells are relaxed, but pressures for solid, in, and out walls are computed within the iteration loop, as these pressures are functions of the pressures in the full cells next to them. Free surface pressures remain the same throughout the time cycle, since they are functions of velocity and viscosity only.
8. VELCTS(K) calculates the velocity field from the general equations for velocities between two full cells, two free surface cells, or a full cell and a free surface cell, then calls BNDCND(K).

9. MOVPAR moves the particles with the current velocities.
10. DENCHG finds the cells which have changed density as a result of a temporary particle movement.
11. REFCEL reflags the cells which have changed, i.e., free surface to empty, destroys and creates particles as needed, calls BNDCND (1), and calculates the free surface pressures. REFCEL also calls FLGCEL for the two-layer model, and FLGCEL flags interface and contributing cells appropriately.

VI. MODELING

Once the computer program has been written, the most important aspect of simulation is the choice of boundary and initial conditions. Thus, care must be taken that boundary conditions be developed that are analogous to physical boundary conditions.

Two models were used for the density stratification, each having advantages and disadvantages. The fluid can be divided into immiscible fluid layers each with a different density. This approach has the advantage that any densities can be assigned to the layers, but if mixing is a significant factor the results will be unrealistic. Calculation time increases with the number of layers used.

An alternate approach is to assign the fluid an arbitrary continuous density. This model takes slightly more calculation time than a two-layer model and requires that local density variations be small. On the other hand, it models a single fluid with variable density quite well.

Noslip and freeslip walls were tried for reservoir problems. With noslip walls, boundary layer build-up never exceeded two cell heights. Since boundary layers are only a few percent of the depth for prototype problems; freeslip walls give a more realistic model for reservoirs less than 100 cells deep. Figure 2 (page 20) shows the modeling for a typical reservoir problem.

VII. PROBLEMS SIMULATED WITH NUMAC

NUMAC was written with two types of nonhomogeneities in mind: (1) Intrusion of one fluid into another. Such problems occur for example in the disposal of wastes. (2) Density stratified flows. These arise naturally in lakes and reservoirs. There is no restriction to these types of density variations, but many problems in oceanography, hydraulics, and meteorology are of these two types. NUMAC is a useful tool for investigating these density phenomena.

A representative oceanographic application is given in Figure 9. A salt water wedge flows into a shallow layer of fresh water. To simulate the sloping beach, the problem was run with a grid parallel to the bottom but with a horizontal gravity component.

Another problem of interest is the motion of a dense block of fluid through a less dense layer under the influence of gravity. This represents the disposal of a pollutant in a river. This sequence is presented in Figure 10.

Figure 11 shows the flow of a bouyant plume into a density stratified tank.

The increased exploitation of the sea requires the development of improved criteria for undersea pipelines and structures. NUMAC is used to show impact pressures on submerged structures to give increased understanding of wave force phenomena. Figure 12 shows a

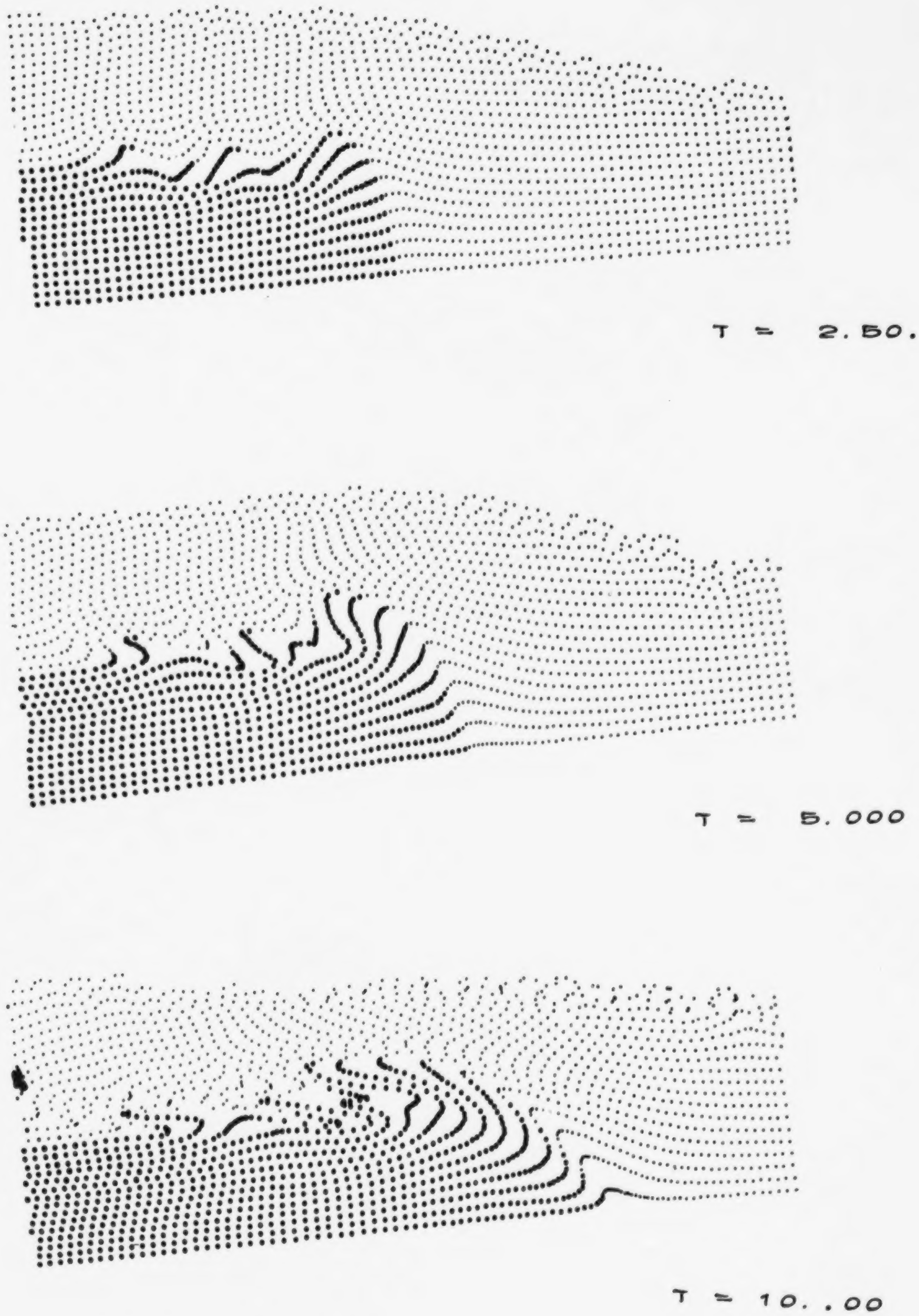


Figure 9. The intrusion of a salt water wedge. The densities were 1 above and 1. 2 below. Viscosity μ = . 0001.

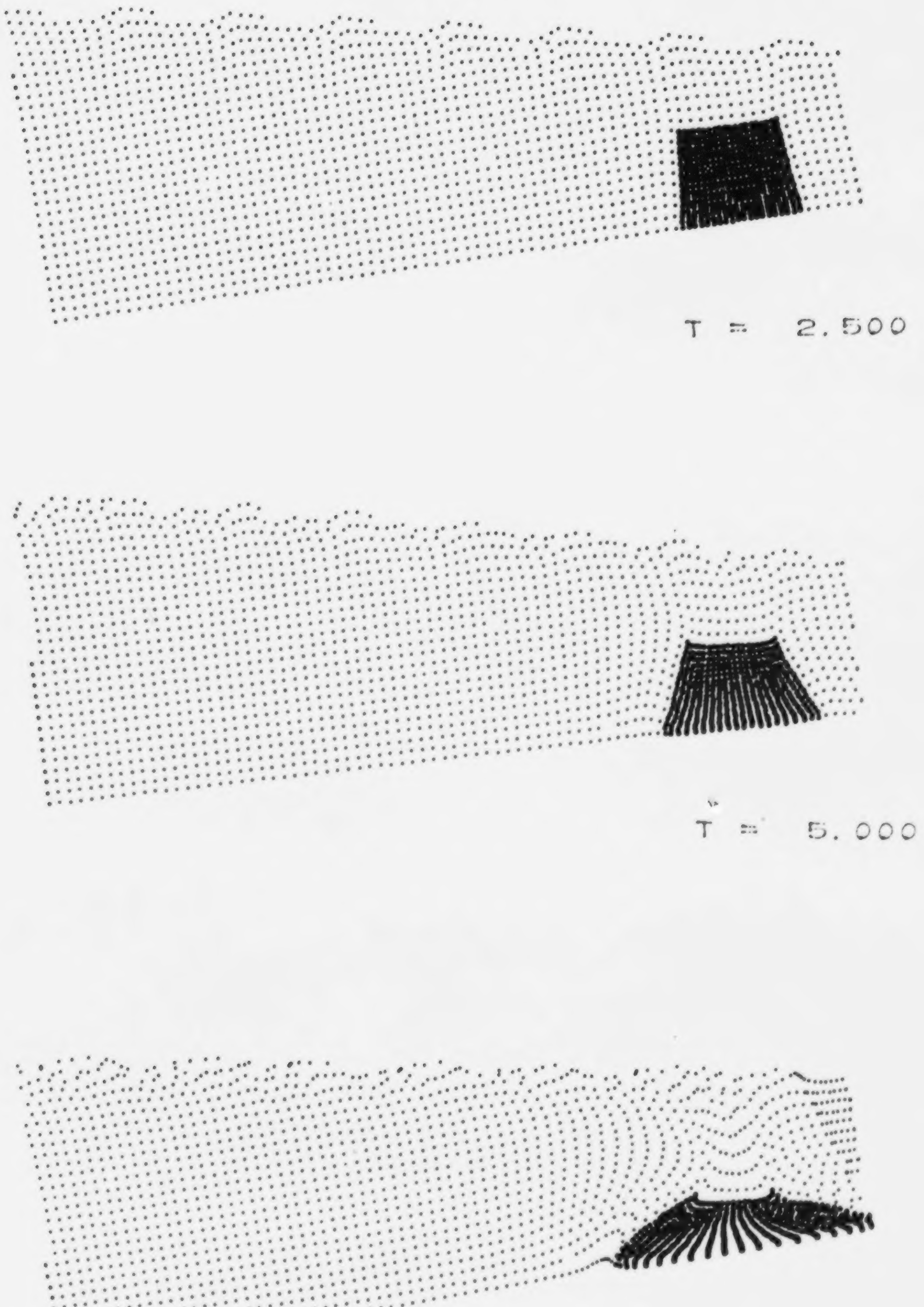


Figure 10. The flow of a denser pollutant. Fluid density is 1, pollutant density is 1.2, viscosity is .0001.

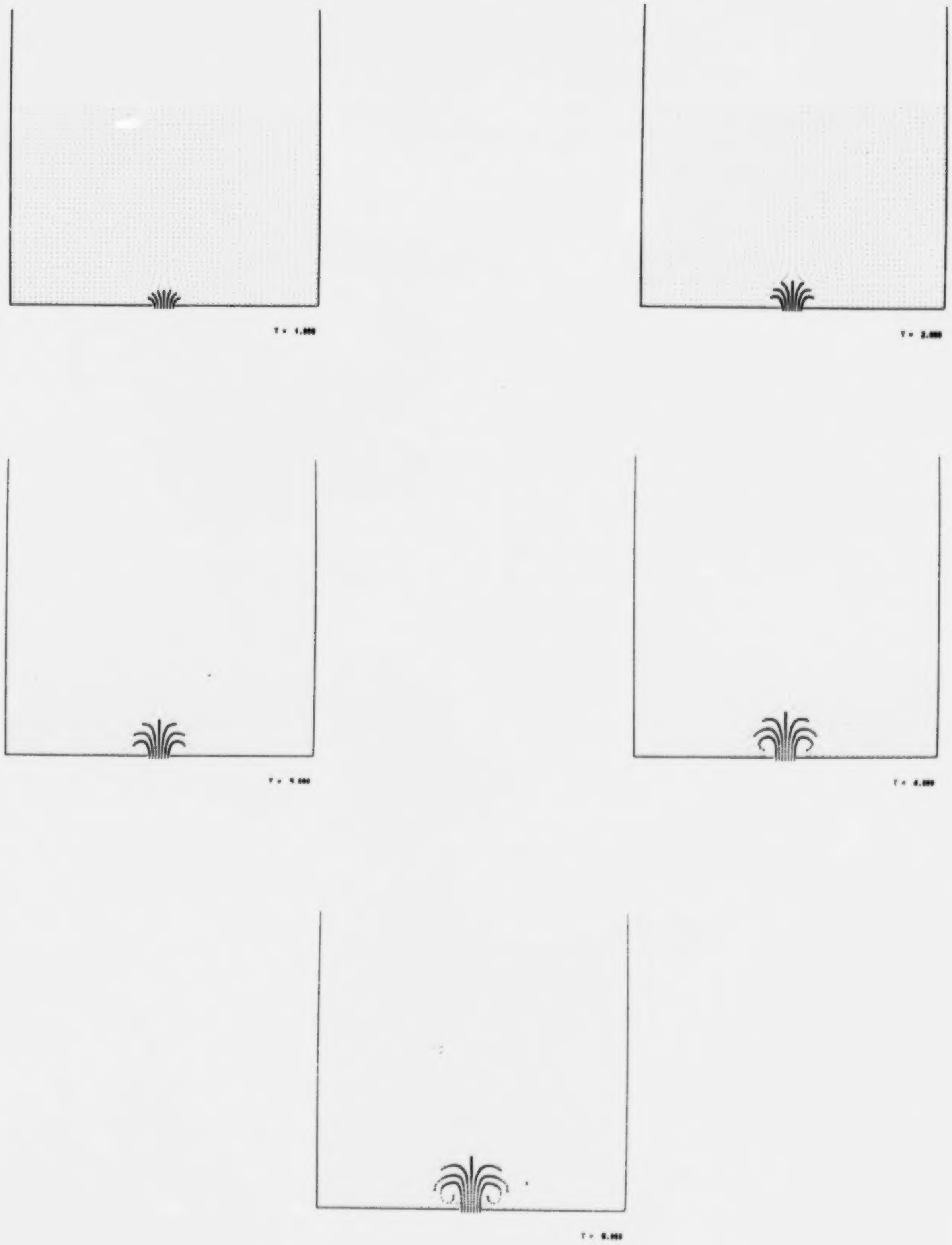

T = 1.000
T = 2.000
T = 3.000
T = 4.000
T = 5.000

Figure 11. Flow into a density stratified tank. The density profile is linear and the incoming fluid has a density that matches one of the tank strata.

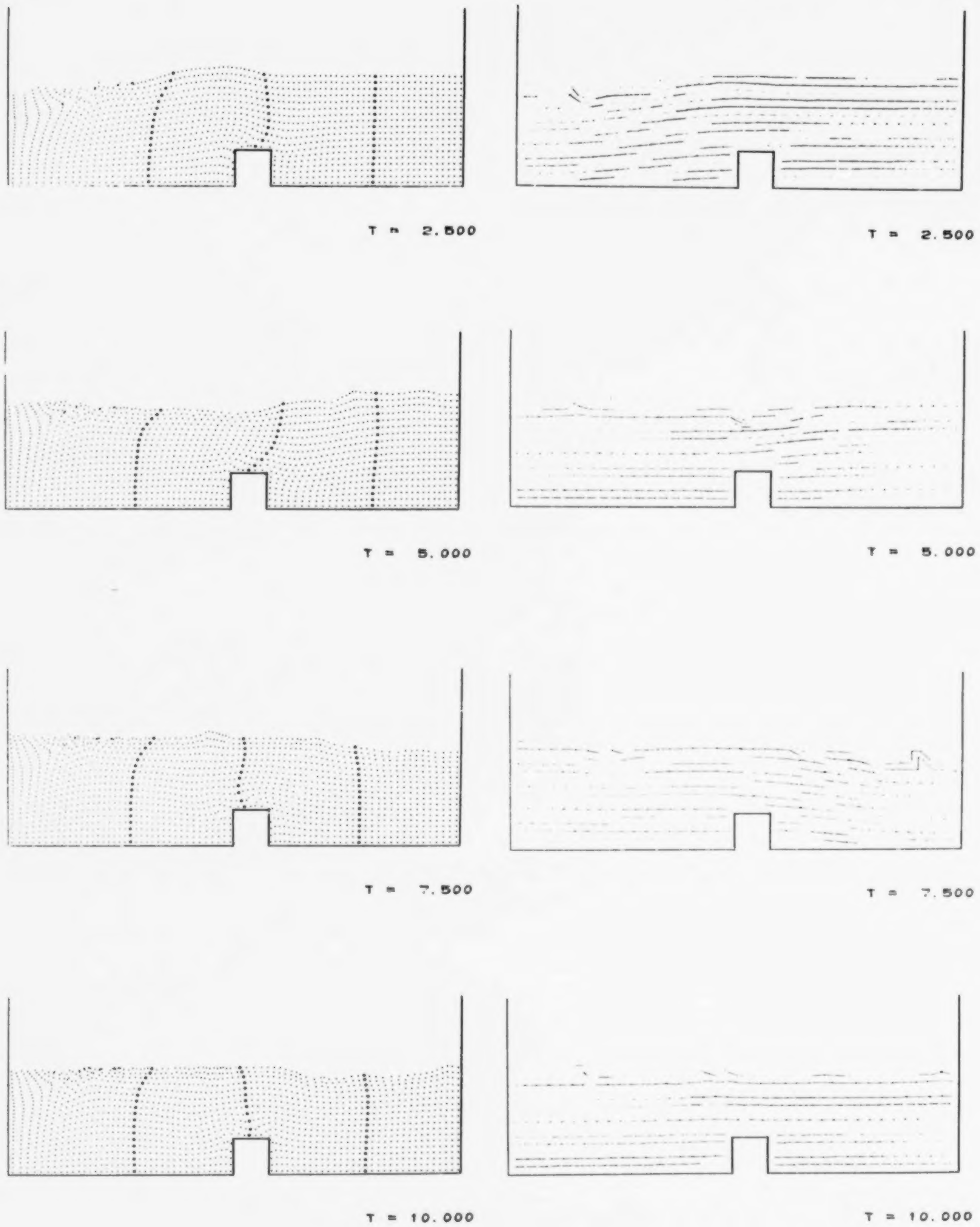


Figure 12. Wave motion over a submerged conduit. On the right are pressure contours.

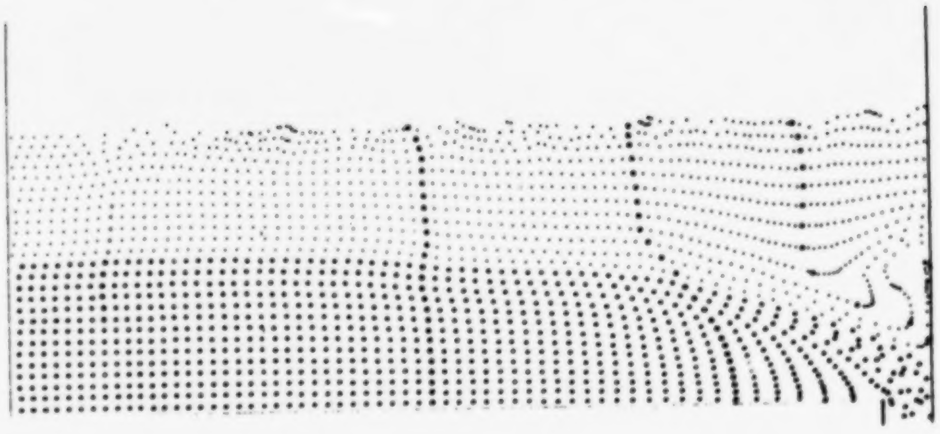
wave passing over a submerged conduit. Accompanying the simulation sequence are the corresponding pressure contour plots.

Withdrawal From a Density Stratified Reservoir

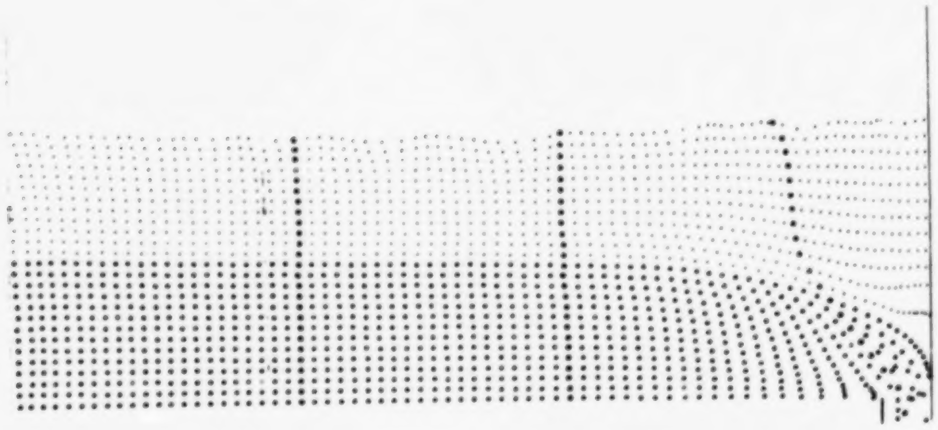
Stratified currents are of engineering interest in such problems as meteorologic disposal of industrial wastes and reservoir sedimentation with selective withdrawal of quality waters. The flow from a density stratified reservoir has been studied in detail.

The research procedure was to simulate a reservoir and investigate the effects of viscosity variation, density stratifications, and the presence of a submerged ridge on the flow pattern. Both the continuous and the two-layered models were used.

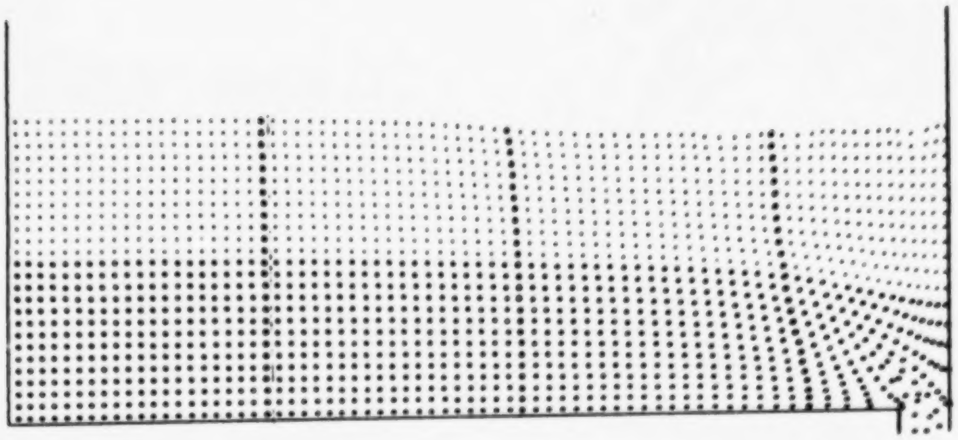
Figures 13 and 14 contain selected frames from two reservoir simulation problems using the two-layered model: withdrawal with and without a submerged ridge. These reservoir problems were normalized so that the following two conditions held. The reference density (in this case the density, ρ_1, of the upper layer) was scaled to unity. Gravity, g', was scaled to unity. Scaling the variables in this way, and using a density in the lower layer $\rho_2 = 1.2$ with a lower layer depth $d_2 = .7$, the normalized upstream steady state velocity in the lower layer approached $U_{\infty 2} = .1$, during machine calculation. Thus, the Froude number for the lower layer



T = 10.000



T = 5.000



T = 2.500

Figure 13. Flow from a two-layered reservoir. The density is 1 in the upper layer, 1.2 in the lower. Viscosity is .0001.

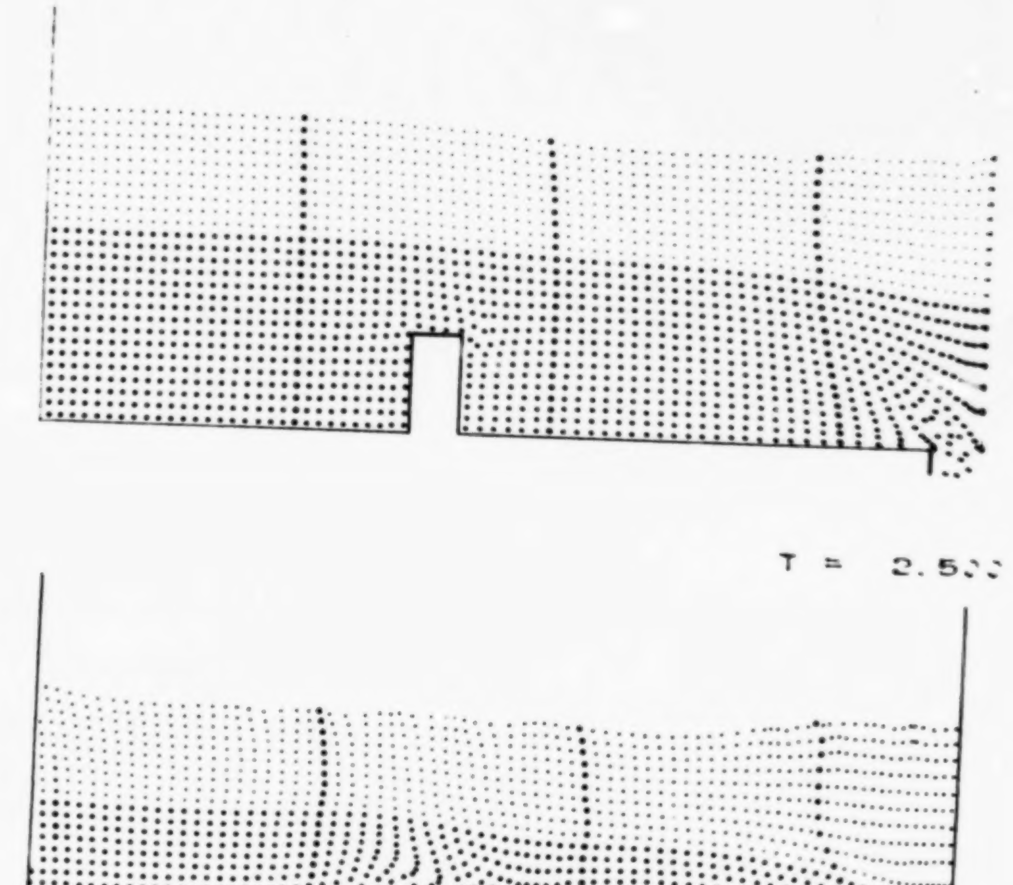


Figure 14. Flow over a submerged ridge. The same densities and viscosities as in Figure 13 were used so the effect of the ridge could be studied.

$$F_2 = \frac{U_{\infty 2}}{\sqrt{g'd_2 \frac{(\rho_2 - \rho_1)}{\rho_2}}} = .293$$

According to Yih (1965), stagnation occurs for $F_2 < 1/\pi$. Vortex formation can be observed in the last two frames in both sequences.

A reservoir with a submerged ridge was also simulated in a fluid with a linear stratification. Figure 15 shows the effect of the ridge. As expected, the ridge hinders withdrawal from the lower strata and blocking forces the contributing layer up.

Figure 16 shows the effect of viscosity in the model. It is seen that for reduced viscosity, velocity is increased uniformly in the fluid.

Figure 17 illustrates the effects of the density gradient in the linearly stratified reservoir. For an increased density gradient the inertial effects are seen to increase the flow from the lower layers.

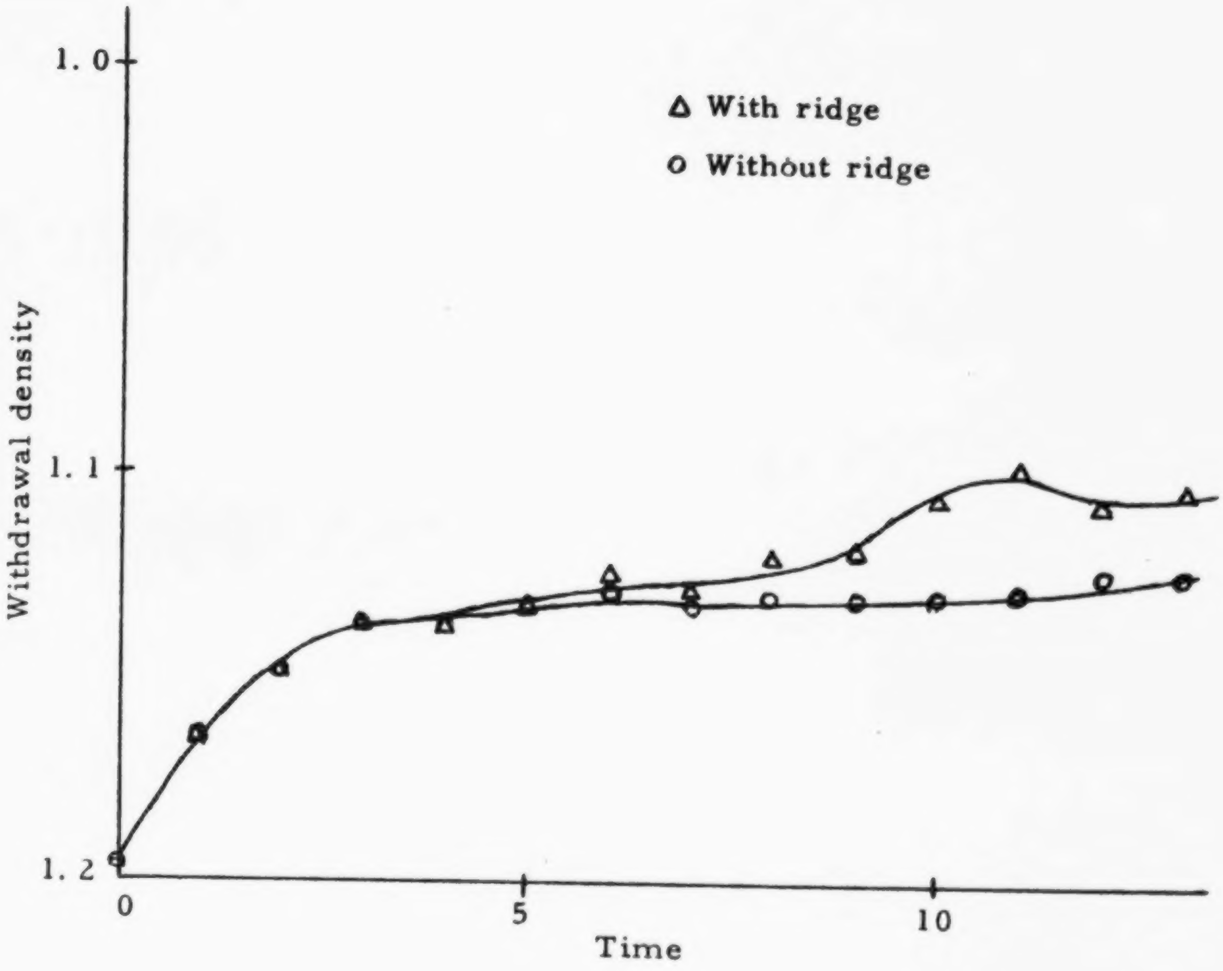
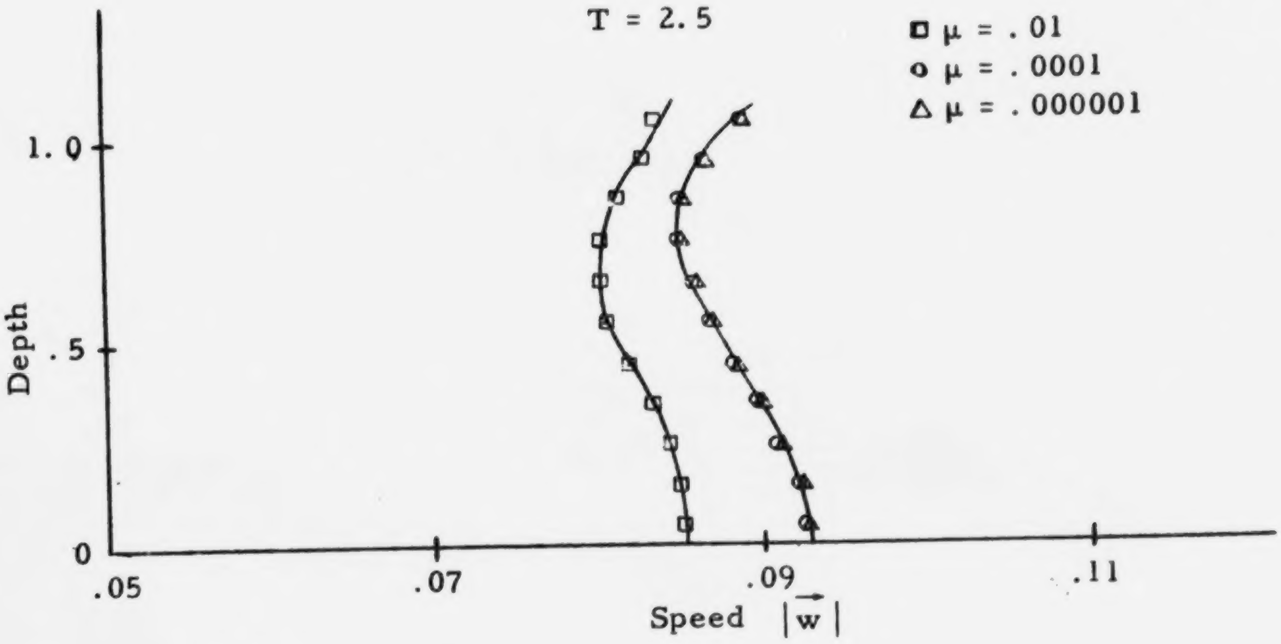


Figure 15. The effects of a submerged ridge. The reservoir was originally stratified linearly with normalized density 1 on the surface and 1. 2 on the bottom.



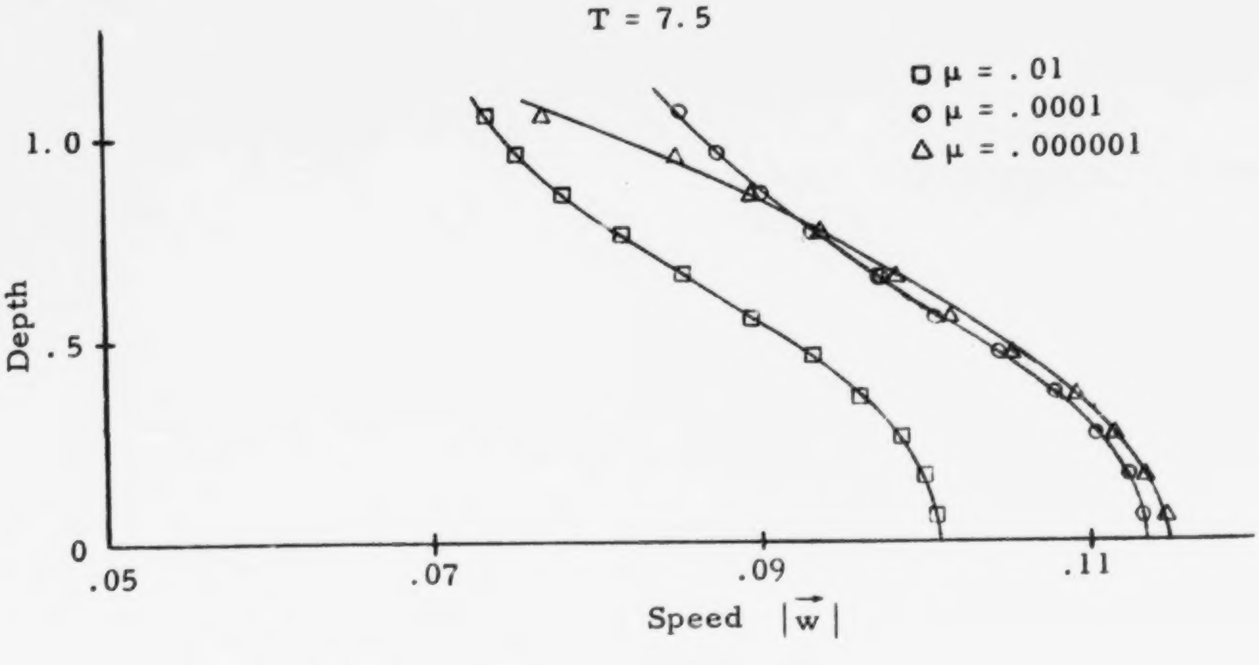


Figure 16. Illustration of viscosity effect on velocity profiles. The profile is one quarter of the model width downstream from the orifice. The normalized density was held constant at $\rho = 1$.

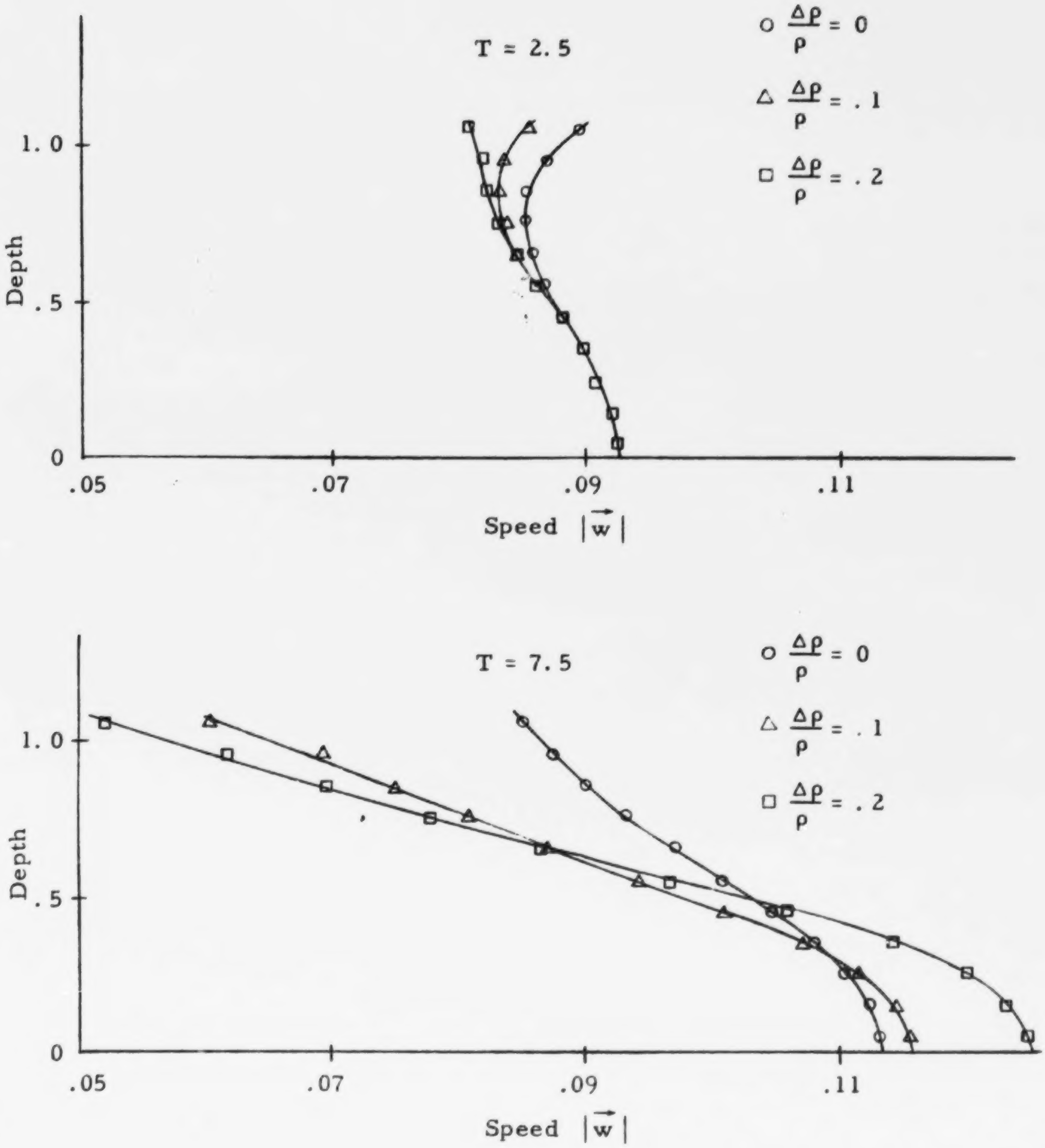


Figure 17. Illustration of the effect of density gradients on velocity profiles. The profile is one quarter of the model width downstream from the orifice. The normalized viscosity, $\mu = .0001$.

VIII. DISCUSSION

The NUMAC method has been presented as a general method for finding the transient flow of a nonhomogeneous, viscous, incompressible fluid. The results in the preceding section were obtained using 800 cells and 3000 particles. The storage requirement was typically 65,000 locations. Using a time step that was near the maximum allowable by the stability conditions, one time cycle took seven seconds on a CDC 6600. A typical run of 200 cycles took twenty-three minutes. It is felt that this size and the running times are nearly minimal. For proper simulation and for problems that are geometrically more complex, more cells and particles should be used. Detail and accuracy are limited only by the size of the machine available.

The current version of NUMAC admits only boundaries that are expressible in terms of the grid, i. e., those which have been "rectangularized." If circular or oblique boundaries are desired, the boundary may be approximated in terms of the grid. Standard techniques in the numerical solution of partial differential equations may then be used to apply to the oblique or circular boundary conditions in the boundary cells.

Similarly, NUMAC has been presented in a two-dimensional form. The method is valid for three dimensions but requires a good deal more programming and quite a bit more storage.

The usefulness of NUMAC in its present form has been demonstrated. Because of its generality, the user will find it a valuable tool in many types of hydrodynamic problems.

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APPENDIX

```
      PROGRAM SPLASH(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,FILM,TAPE99=
     $                FILM,TAPE98,TAPE7)
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,IPEF,ITAL,TWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
C
C
C     PROGRAM SPLASH CONTAINS THE TWO-FLUID EXTENSION OF THE MAC METHOD.
C
C     THIS PROGRAM WAS WRITTEN IN JULY OF 1967 BY MICHAEL D. TERRY.
C
C
      MI=5
      MO=6
      NTP=7
      REWIND NTP
      KD=1
      ITV=6LFILM
      LI=0
      READ (MI,1000) NB1,NB2,NI,NJ,NP,NPR,IVO
      READ (MI,1000) ISAVE,IWDG,NC,ISPACE
      READ (MI,1001) W,H,GX,GY,UO,VO,DT
      READ (MI,1001) MU1,MU2,R1,R2,TL,DELTAS,DTP
      READ (MI,1001) DTCP,DTPP,DTVP,DTPR,DXP,DYP
      READ (MI,1002) (NF(I),XPO(I),YPO(I),XPL(I),YPL(I),UPO(I),VPO(I),
     $      I=1,NPR)
      IF (ISAVE.GT.0) GO TO 106
      DXC=W/(NI-2)
      DYC=H/(NJ-2)
      WRITE (MO,1003)
      WRITE (MO,1004) NB1,NB2,NI,NJ,NP,NPR,IVO
      WRITE (MO,1012)
      WRITE (MO,1004) ISAVE,IWDG,NC,ISPACE
      WRITE (MO,1005)
      WRITE (MO,1006) W,H,GX,GY,UO,VO,DT
      WRITE (MO,1007)
      WRITE (MO,1006) MU1,MU2,R1,R2,TL,DELTAS,DTP
      WRITE (MO,1008)
      WRITE (MO,1006) DTCP,DTPP,DTVP,DTPR,DXP,DYP
      WRITE (MO,1009)
      DO 105 I=1,NPR
  105 WRITE (MO,1010) NF(I),XPO(I),YPO(I),XPL(I),YPL(I),UPO(I),VPO(I)
      WRITE (MO,1013)
      WRITE (MO,1006) DXC,DYC
```

```
      NIM1=NI-1
      NIP1=NI+1
      NJM1=NJ-1
      NJP1=NJ+1
      DELTAS=DELTAS*.5
      G=SQRT(GX*GX+GY*GY)
      GXD=GX*DXC
      GYD=GY*DYC
      ODX=1./DXC
      ODY=1./DYC
      DXCD2=DXC*.5
      DYCD2=DYC*.5
      DXI4=4.*ODX
      DYI4=4.*ODY
      IF (G.EQ.0) GO TO 197
      GH=1./(G*H)
      GO TO 198
  197 GH=1./H
  198 DTDX=DT*ODX
      DTDY=DT*ODY
      DT2=2.*DT
      ODXS=ODX*ODX
      ODYS=ODY*ODY
      ODX2=2.*ODX
      ODY2=2.*ODY
      DTDXS=DT2*ODXS
      DTDYS=DT2*ODYS
      GXDT=GX*DT
      GYDT=GY*DT
      DT4DX=DTDX*.25
      DT4DY=DTDY*.25
      DXO2DY=DXCD2*ODY
      DYO2DX=DYCD2*ODX
      DXODY=DXC*ODY
      DYODX=DYC*ODX
      MSK(1)=7
      IMP(1)=1
      DV(1)=1
      MKC(1)=.NOT.MSK(1)
      DO 100 I=2,14
      MSK(I)=MSK(I-1)*8
      MKC(I)=.NOT.MSK(I)
      IMP(I)=IMP(I-1)*8
  100 DV(I)=DV(I-1)/8
      T=0.
      XC(1)=-DXCD2
      YC(1)=-DYCD2
      DO 101 I=2,NI
  101 XC(I)=XC(I-1)+DXC
      DO 102 J=2,NJ
  102 YC(J)=YC(J-1)+DYC
      DO 1 J=1,NJ
      DO 1 I=1,NI
      A(I,J)=0.
      B1(I,J)=0.
      B2(I,J)=0.
      B3(I,J)=0.
      B4(I,J)=0.
      CS(I,J)=0
      MU(I,J)=0.
      NK(I,J)=0.
      NKT(I,J)=0.
```

NOT REPRODUCIBLE

```
      P(I,J)=0.
      R(I,J)=0.
      SR(I,J)=0.
    1 SRT(I,J)=0.
      DO 2 J=1,NJP1
      DO 2 I=1,NIP1
      PSI(I,J)=0.
      U(I,J)=0.
      UT(I,J)=0.
      V(I,J)=0.
      VT(I,J)=0.
    2 ZET(I,J)=0.
      DO 3 K=1,NP
      PS(K)=0
      UP(K)=0.
      VP(K)=0.
      XP(K)=0.
    3 YP(K)=0.
      DO 104 J=1,NJ
      DO 104 I=1,NI
      DO 103 K=3,6
  103 CS(I,J)=IMP(K).OR.(CS(I,J).AND.MKC(K))
      CS(I,J)=6.OR.(CS(I,J).AND.MKC)
      KT=5*IMP(2)
  104 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(2))
      CALL CELSET
      CALL PARSET
      CALL DENVIS
      CALL BNDCND(3)
  106 CALL CNTROL
      IF (KD.EQ.2) GO TO 113
      IC=0
      ICNTR=0
      LLL=0
      CALL QNTPRS(1)
  107 DO 108 J=1,NJ
      DO 108 I=1,NI
  108 CS(I,J)=IMP(13).OR.(CS(I,J).AND.MKC(13))
      ITER=0
  109 ITER=ITER+1
      ICNTR=ICNTR+1
      ITEST=4
      EPS=.0008
      CALL PRSITN
      IF (KD.EQ.2) GO TO 106
      DO 199 J=1,NJ
      DO 199 I=1,NI
      NKT(I,J)=0.
  199 SPT(I,J)=0.
      CALL VELCTS(2)
      KKK=1
      CALL MOVPAR
      IF (KD.EQ.2) GO TO 106
      CALL DENCHG
      IF (LL.EQ.0) GO TO 112
      IF (LL.GT.9) GO TO 202
      IF (LLL.LE.9) GO TO 200
      IC=0
      GO TO 201
  200 IC=IC+1
      IF (IC.GT.10) GO TO 112
  201 LLL=LL
  202 CALL QNTPRS(2)
      IF (ITER.NE.3) GO TO 109
      DO 111 J=2,NJM1
      DO 111 I=2,NIM1
      K11=CS(I,J).AND.MSK(11)
      K11=K11*DV(11)
      IF (K11.EQ.2) GO TO 110
      K10=CS(I,J).AND.MSK(10)
      K10=K10*DV(10)
      IF (K10.NE.2) GO TO 111
      K13=CS(I,J).AND.MSK(13)
      K13=K13*DV(13)
      IF (K13.NE.1) GO TO 111
      CS(I,J)=IMP(10).OR.(CS(I,J).AND.MKC(10))
      GO TO 111
  110 K13=CS(I,J).AND.MSK(13)
      K13=K13*DV(13)
      IF (K13.NE.1) GO TO 111
      CS(I,J)=IMP(11).OR.(CS(I,J).AND.MKC(11))
  111 CONTINUE
      GO TO 107
  112 ITEST=13
      EPS=.0002
      WRITE (MO,1011)
      CALL PRSITN
      IF (KD.EQ.2) GO TO 106
      CALL VELCTS(1)
      KKK=2
      CALL MOVPAR
      IF (KD.EQ.2) GO TO 106
      CALL DENVIS
      IF (KD.EQ.2) GO TO 106
      CALL REFCEL
      GO TO 106
  113 CALL TVEND
      STOP
 1000 FORMAT(7I10)
 1001 FORMAT(7F10.0)
 1002 FORMAT(I10,6F10.0)
 1003 FORMAT(1H1,10X49HMAC METHOD SOLUTION OF TWO-MATERIAL FLUID PROBLEM
     $///1H-,10X5HINPUT/1H-,22X3HNR1,12X3HNR2,13X2HNI,13X2HNJ,13X2HNP,12
     $X3HNPR,12X3HIVS)
 1004 FORMAT(1H ,10X,7I15)
 1005 FORMAT(1H0,24X1HW,14X1HH,13X2HGX,13X2HGY,13X2HU0,13X2HV0,13X2HDT)
 1006 FORMAT(1H ,10X,7F15.8)
 1007 FORMAT(1H0,22X3HMU1,12X3HMU2,11X4HRHO1,11X4HRHO2,13X2HTL,9X6HDELTA
     $S,12X3HDTP)
 1008 FORMAT(1H0,21X4HDTCP,11X4HDTPP,11X4HDTVP,11X4HDTPR,12X3HDXP,12X3HD
     $YP)
 1009 FORMAT(1H0,23X2HNF,12X3HXP0,12X3HYP0,12X3HXPL,12X3HYPL,12X3HUP0,12
     $X3HVP0)
 1010 FORMAT(1H ,10X,I15,6F15.8)
 1011 FORMAT(1H0)
 1012 FORMAT(1H0,20X5HISAVE,11X4HIWDG,13X2HNC,9X6HISPACE)
 1013 FORMAT(1H0,22X3HDXC,12X3HDYC)
      END
      SUBROUTINE CELSET
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPFF,ITAL,TWINK,INTS,IPT,IUP
      COMMON/DELTAS/DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
```

NOT REPRODUCIBLE

```
$        DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
 COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
$        MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
$        ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
$        VO,W,IVC,DXO2DY,DYO2DX,DXODY,DYODX
 COMMON A(40,20),A1(40,20),A2(40,20),A3(40,20),B4(40,20),CS(40,20),
$        DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
$        NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
$        SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
$        VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
$        YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
 COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
$        XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
 REAL MU,MU1,MU2,NK,NKT
 INTEGER CS,PS
 DIMENSION IA(100),TYPF(20),XB(22),YB(22)
 INTEGER TYPE
 PIMIN=0.
 PIMAX=0.
 PJMIN=0.
 PJMAX=0.
 DO 300 I=1,NI
 PIMIN=AMIN1(XC(I)-DXC,PIMIN)
300 PIMAX=AMAX1(XC(I)+DXC,PIMAX)
 DO 301 J=1,NJ
 PJMIN=AMIN1(YC(J)-DYC,PJMIN)
301 PJMAX=AMAX1(YC(J)+DYC,PJMAX)
 IF (IWDG.LE.0) GO TO 302
 PLMIN=-.5
 PLMAX=4.0625
 GO TO 303
302 PLMIN=AMIN1(PIMIN,PJMIN)
 PLMAX=AMAX1(PIMAX,PJMAX)
303 XMIN=PLMIN
 XMAX=PLMAX
 XI=PLMIN
 XA=PLMAX
 NB=NB1
 KNBP=1
 NBP=NB+1
200 READ (MI,2000) (XB(I),YB(I),I=1,NBP)
 READ (MI,2001) (TYPE(I),I=1,NB)
 XB(NBP+1)=XB(2)
 YB(NBP+1)=YB(2)
 DO 100 M=1,NB
 IF (IWDG.LE.0) GO TO 304
 XS=(5.*XB(M)-YB(M))/5.2
 YS=(YB(M)+.2*XB(M))/1.04
 XF=(5.*XB(M+1)-YB(M+1))/5.2
 YF=(YB(M+1)+.2*XB(M+1))/1.04
 GO TO 305
304 XS=XB(M)
 YS=YB(M)
 XF=XB(M+1)
 YF=YB(M+1)
305 IF (XB(M)-XB(M+1)) 201,215,208
201 I=XB(M)*ODX+2.001
 J=YB(M)*ODY+1.999
202 CS(I,J)=2.OR.(CS(I,J).AND.MKC)
 KT=TYPE(M)*IMP(2)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(2))
 KT=2*IMP(4)
```

```
 CS(I,J+1)=KT.OR.(CS(I,J+1).AND.MKC(4))
 KT=3*IMP(7)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(7))
 KT=3*IMP(8)
 CS(I,J+1)=KT.OR.(CS(I,J+1).AND.MKC(8))
204 I=I+1
 IF (XC(I) .LE. XB(M+1)) 202,205
205 IF (YB(M+2)-YB(M+1)) 206,236,207
206 KT=2*IMP(5)
 CS(I-1,J)=KT.OR.(CS(I-1,J).AND.MKC(5))
 GO TO 236
207 CS(I,J)=1.OR.(CS(I,J).AND.MKC)
 KT=3*IMP(7)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(7))
 GO TO 231
208 I=XB(M)*ODX+1.999
 J=YB(M)*ODY+2.001
209 CS(I,J)=2.OR.(CS(I,J).AND.MKC)
 KT=TYPE(M)*IMP(2)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(2))
 KT=2*IMP(4)
 CS(I,J-1)=KT.OR.(CS(I,J-1).AND.MKC(4))
 KT=4*IMP(7)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(7))
 KT=4*IMP(8)
 CS(I,J-1)=KT.OR.(CS(I,J-1).AND.MKC(8))
211 I=I-1
 IF (XC(I).GE.XB(M+1)) 209,212
212 IF (YB(M+2)-YB(M+1)) 214,236,213
213 KT=2*IMP(5)
 CS(I+1,J)=KT.OR.(CS(I+1,J).AND.MKC(5))
 GO TO 236
214 CS(I,J)=1.OR.(CS(I,J).AND.MKC)
 KT=4*IMP(7)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(7))
 GO TO 231
215 IF (YB(M).LT.YB(M+1)) 216,223
216 I=XB(M)*ODX+2.001
 J=YB(M)*ODY+2.001
217 CS(I,J)=2.OR.(CS(I,J).AND.MKC)
 KT=TYPF(M)*IMP(2)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(2))
 KT=2*IMP(4)
 CS(I-1,J)=KT.OR.(CS(I-1,J).AND.MKC(4))
 KT=2*IMP(7)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(7))
 KT=2*IMP(8)
 CS(I-1,J)=KT.OR.(CS(I-1,J).AND.MKC(8))
219 J=J+1
 IF (YC(J).LE.YB(M+1)) 217,220
220 IF (XB(M+2)-XB(M+1)) 222,236,221
221 KT=2*IMP(5)
 CS(I,J-1)=KT.OR.(CS(I,J-1).AND.MKC(5))
 GO TO 236
222 CS(I,J)=1.OR.(CS(I,J).AND.MKC)
 KT=2*IMP(7)
 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(7))
 GO TO 231
223 I=XB(M)*ODX+1.999
 J=YB(M)*ODY+1.999
224 CS(I,J)=2.OR.(CS(I,J).AND.MKC)
 KT=TYPF(M)*IMP(2)
```

```
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(2))
      KT=2*IMP(4)
      CS(I+1,J)=KT.OR.(CS(I+1,J).AND.MKC(4))
      CS(I,J)=IMP(7).OR.(CS(I,J).AND.MKC(7))
      CS(I+1,J)=IMP(8).OR.(CS(I+1,J).AND.MKC(8))
  226 J=J-1
      IF (YC(J).GE.YB(M+1)) 224,227
  227 IF (M.LT.NB) 228,236
  228 IF (XB(M+2)-XB(M+1)) 229,236,230
  229 KT=2*IMP(5)
      CS(I,J+1)=KT.OR.(CS(I,J+1).AND.MKC(5))
      GO TO 236
  230 CS(I,J)=1.OR.(CS(I,J).AND.MKC)
      CS(I,J)=IMP(7).OR.(CS(I,J).AND.MKC(7))
  231 IF (M.LT.NB) 232,233
  232 KMP1=M+1
      GO TO 234
  233 KMP1=1
  234 IF (TYPE(M).EQ.3.OR.TYPE(KMP1).EQ.3) 235,236
  235 KT=3*IMP(2)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(2))
  236 IF (TYPE(M).EQ.2) GO TO 98
      IF (KNBP.EQ.2) GO TO 97
      XB1(M)=XS
      XB1(M+1)=XF
      YB1(M)=YS
      YB1(M+1)=YF
      XX1(M)=0.
      GO TO 100
   97 XB2(M)=XS
      XB2(M+1)=XF
      YB2(M)=YS
      YB2(M+1)=YF
      XX2(M)=0.
      GO TO 100
   98 IF (KNBP.EQ.2) GO TO 99
      XX1(M)=1.
      GO TO 100
   99 XX2(M)=1.
  100 CONTINUE
      IF (NB2.NE.0) 237,238
  237 NB=NB2
      NB2=0
      KNBP=2
      NBP2=NB+1
      GO TO 200
  238 KODE=1
      DO 264 J=1,NJ
      DO 264 I=1,NI
      K1=CS(I,J).AND.MSK
      K2=CS(I,J).AND.MSK(2)
      K2=K2*DV(2)
      K4=CS(I,J).AND.MSK(4)
      K4=K4*DV(4)
      IF (K2.NE.1) GO TO 258
  239 IF (I.EQ.1) 240,243
  240 L=1
  241 L1=1
      K2A=CS(I+1,J-1).AND.MSK(2)
      K2A=K2A*DV(2)
      IF (K2A.EQ.4) GO TO 255
  242 K2B=CS(I+1,J+1).AND.MSK(2)
      K2B=K2B*DV(2)
      GO TO 253
  243 IF (I.EQ.NI) 244,247
  244 L=1
  245 L1=2
      K2C=CS(I-1,J-1).AND.MSK(2)
      K2C=K2C*DV(2)
      IF (K2C.EQ.4) GO TO 255
  246 K2B=CS(I-1,J+1).AND.MSK(2)
      K2B=K2B*DV(2)
      GO TO 253
  247 IF(J.EQ.1) 248,250
  248 L=1
      K2D=CS(I-1,J+1).AND.MSK(2)
      K2D=K2D*DV(2)
      IF (K2D.EQ.4) GO TO 255
  249 K2B=CS(I+1,J+1).AND.MKC(2)
      K2B=K2B*DV(2)
      GO TO 253
  250 IF (J.EQ.NJ) 251,254
  251 L=1
      K2E=CS(I-1,J-1).AND.MSK(2)
      K2E=K2E*DV(2)
      IF (K2E.EQ.4) GO TO 255
  252 K2B=CS(I+1,J-1).AND.MSK(2)
      K2B=K2B*DV(2)
  253 IF (K2B.EQ.4) GO TO 255
      GO TO 256
  254 L=2
      GO TO 241
  255 KT=2*IMP(3)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(3))
  256 IF (L.EQ.1) GO TO 258
  257 IF (L1.EQ.1) GO TO 245
  258 IF (KODE.EQ.2) GO TO 262
  259 IF (K4.EQ.2) GO TO 262
  260 KODE=1
      IF (K1.EQ.2) GO TO 264
  261 CS(I,J)=1.OR.(CS(I,J).AND.MKC)
      GO TO 264
  262 KODE=1
      IF (K1.EQ.2) GO TO 264
  263 KODE=2
      CS(I,J)=3.OR.(CS(I,J).AND.MKC)
  264 CONTINUE
      DO 266 J=1,NJ
      DO 266 I=1,NI
      K5=CS(I,J).AND.MSK(5)
      K5=K5*DV(5)
      IF (K5.EQ.1) GO TO 266
      K41=CS(I-1,J).AND.MSK(4)
      K41=K41*DV(4)
      IF (K41.EQ.2) GO TO 265
      CS(I,J)=IMP(7).OR.(CS(I,J).AND.MKC(7))
      GO TO 266
  265 KT=2*IMP(7)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(7))
  266 CONTINUE
      RETURN
 2000 FORMAT(7F10.0)
 2001 FORMAT(7I10)
      END
      SUBROUTINE PARSET
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,TWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXT4,DXIN,DXP,DYC,DYCD2,DYT4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DO 299 J=1,NJ
      DO 299 I=1,NI
  299 NK(I,J)=0.
      DO 300 K=1,NP
  300 PS(K)=3.OR.(PS(K).AND.MKC)
      K=1
      DO 301 J=1,NJ
      DO 301 I=1,NI
      KT=4*IMP(9)
  301 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(9))
      TEMPX=DXP
      TEMPY=DYP
      DO 315 II=1,NPR
      DXP=TEMPX
      DYP=TEMPY
      IF (ISPACE.LE.0) GO TO 900
      IF (NF(II).EQ.1) GO TO 900
      DXP=DXP*.5
      DYP=DYP*.5
  900 CONTINUE
      Y=YPO(II)
  302 X=XPO(II)
  303 I=X*ODX+2.
      J=Y*ODY+2.
      K1=CS(I,J).AND.MSK
      K2=CS(I,J).AND.MSK(2)
      K2=K2*DV(2)
      IF (K1.EQ.1) GO TO 308
      IF (K1.EQ.2) GO TO 310
  304 CS(I,J)=4.OR.(CS(I,J).AND.MKC)
      PS(K)=1.OR.(PS(K).AND.MKC)
  400 KT=NF(II)*IMP(3)
      PS(K)=KT.OR.(PS(K).AND.MKC(3))
      KT=IMP(4)
      PS(K)=KT.OR.(PS(K).AND.MKC(4))
      NK(I,J)=NK(I,J)+1.
      K9=CS(I,J).AND.MSK(9)
      K9=K9*DV(9)
      IF (K9.NE.4) GO TO 305
      KT=NF(II)*IMP(9)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(9))
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      GO TO 306
  305 IF (NF(II).EQ.K9) GO TO 306
      KT=3*IMP(9)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(9))
  306 UT(I,J)=UPO(II)
      VT(I,J)=VPO(II)
      UT(I+1,J)=UPO(II)
      VT(I,J+1)=VPO(II)
  307 XP(K)=X
      YP(K)=Y
      K=K+1
  308 X=X+DXP
      IF (X.GT.XPL(II)) 309,303
  309 Y=Y+DYP
      IF (Y.GT.YPL(II)) 315,302
  310 IF (K2.NE.1) GO TO 308
  311 PS(K)=2.OR.(PS(K).AND.MKC)
      K7=CS(I,J).AND.MSK(7)
      K7=K7*DV(7)
      KT=K7*IMP(2)
      PS(K)=KT.OR.(PS(K).AND.MKC(2))
      GO TO 400
  315 CONTINUE
      DO 316 J=1,NJ
      DO 316 I=1,NI
      CS(I,J)=IMP(10).OR.(CS(I,J).AND.MKC(10))
  316 CS(I,J)=IMP(11).OR.(CS(I,J).AND.MKC(11))
      KT=IMP(5)
      DO 317 K=1,NP
  317 PS(K)=KT.OR.(PS(K).AND.MKC(5))
      KT=2*IMP(5)
      DO 319 K=1,NP
      IF (XP(K).GT.1.024.AND.XP(K).LT.1.026) GO TO 318
      IF (XP(K).GT.2.024.AND.XP(K).LT.2.026) GO TO 318
      IF (XP(K).GT.3.024.AND.XP(K).LT.3.026) GO TO 318
      GO TO 319
  318 PS(K)=KT.OR.(PS(K).AND.MKC(5))
  319 CONTINUE
      CALL REFCEL
      RETURN
      END
      SUBROUTINE FLGCEL
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,TWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXT4,DXIN,DXP,DYC,DYCD2,DYT4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DO 818 J=2,NJM1
      JM1=J-1
      JP1=J+1
      DO 818 I=2,NIM1
      IM1=I-1
      IP1=I+1
      K1=CS(I,J).AND.MSK
      IF (K1.EQ.4) GO TO 800
      IF (K1.NE.5) GO TO 818
  800 K9=CS(I,J).AND.MSK(9)
      K9=K9*DV(9)
      IF (K9.NE.3) GO TO 815
    1 KT1=2*IMP(11)
      KT2=2*IMP(10)
      CS(I,J)=KT1.OR.(CS(I,J).AND.MKC(11))
      CS(I,JP1)=KT1.OR.(CS(I,JP1).AND.MKC(11))
      CS(IP1,JP1)=KT1.OR.(CS(IP1,JP1).AND.MKC(11))
      CS(IP1,J)=KT1.OR.(CS(IP1,J).AND.MKC(11))
      CS(IP1,JM1)=KT1.OR.(CS(IP1,JM1).AND.MKC(11))
      CS(I,JM1)=KT1.OR.(CS(I,JM1).AND.MKC(11))
      CS(IM1,JM1)=KT1.OR.(CS(IM1,JM1).AND.MKC(11))
      CS(IM1,J)=KT1.OR.(CS(IM1,J).AND.MKC(11))
      CS(IM1,JP1)=KT1.OR.(CS(IM1,JP1).AND.MKC(11))
      CS(I,J)=KT2.OR.(CS(I,J).AND.MKC(10))
      CS(I,JP1)=KT2.OR.(CS(I,JP1).AND.MKC(10))
      CS(IP1,JP1)=KT2.OR.(CS(IP1,JP1).AND.MKC(10))
      CS(IP1,J)=KT2.OR.(CS(IP1,J).AND.MKC(10))
      CS(IP1,JM1)=KT2.OR.(CS(IP1,JM1).AND.MKC(10))
      CS(I,JM1)=KT2.OR.(CS(I,JM1).AND.MKC(10))
      CS(IM1,JM1)=KT2.OR.(CS(IM1,JM1).AND.MKC(10))
      CS(IM1,J)=KT2.OR.(CS(IM1,J).AND.MKC(10))
      CS(IM1,JP1)=KT2.OR.(CS(IM1,JP1).AND.MKC(10))
      ML=1
      MR=1
      MB=1
      MT=1
      K1A=CS(IM1,J).AND.MSK
      IF (K1A.EQ.2) ML=2
      K1B=CS(IP1,J).AND.MSK
      IF (K1B.EQ.2) MR=2
      K1C=CS(I,JM1).AND.MSK
      IF (K1C.EQ.2) MB=2
      K1D=CS(I,JP1).AND.MSK
      IF (K1D.EQ.2) MT=2
      IF (ML+MR+MB+MT.NE.4) GO TO 801
      JM2=J-2
      JP2=J+2
      IM2=I-2
      IP2=I+2
    2 KT2=2*IMP(10)
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      CS(IP2,JP2)=KT2.OR.(CS(IP2,JP2).AND.MKC(10))
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      CS(IP2,JM2)=KT2.OR.(CS(IP2,JM2).AND.MKC(10))
      CS(IP1,JM2)=KT2.OR.(CS(IP1,JM2).AND.MKC(10))
      CS(I,JM2)=KT2.OR.(CS(I,JM2).AND.MKC(10))
      CS(IM1,JM2)=KT2.OR.(CS(IM1,JM2).AND.MKC(10))
      CS(IM2,JM2)=KT2.OR.(CS(IM2,JM2).AND.MKC(10))
      CS(IM2,JM1)=KT2.OR.(CS(IM2,JM1).AND.MKC(10))
      CS(IM2,J)=KT2.OR.(CS(IM2,J).AND.MKC(10))
      CS(IM2,JP1)=KT2.OR.(CS(IM2,JP1).AND.MKC(10))
      CS(IM2,JP2)=KT2.OR.(CS(IM2,JP2).AND.MKC(10))
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
      GO TO 818
  801 IF (ML+MR+MB.NE.6) GO TO 802
      JP2=J+2
    3 KT2=2*IMP(10)
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
      GO TO 818
  802 IF (ML+MR+MT.NE.6) GO TO 803
      JM2=J-2
    4 KT2=2*IMP(10)
      CS(I,JM2)=KT2.OR.(CS(I,JM2).AND.MKC(10))
      CS(IP1,JM2)=KT2.OR.(CS(IP1,JM2).AND.MKC(10))
      CS(IM1,JM2)=KT2.OR.(CS(IM1,JM2).AND.MKC(10))
      GO TO 818
  803 IF (ML+MB+MT.NE.6) GO TO 804
      IP2=I+2
    5 KT2=2*IMP(10)
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      GO TO 818
  804 IF (MR+MB+MT.NE.6) GO TO 805
      IM2=I-2
    6 KT2=2*IMP(10)
      CS(IM2,JM1)=KT2.OR.(CS(IM2,JM1).AND.MKC(10))
      CS(IM2,J)=KT2.OR.(CS(IM2,J).AND.MKC(10))
      CS(IM2,JP1)=KT2.OR.(CS(IM2,JP1).AND.MKC(10))
      GO TO 818
  805 IF (ML+MR.NE.4) GO TO 806
      JM2=J-2
      JP2=J+2
    7 KT2=2*IMP(10)
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      CS(IM1,JM2)=KT2.OR.(CS(IM1,JM2).AND.MKC(10))
      CS(I,JM2)=KT2.OR.(CS(I,JM2).AND.MKC(10))
      CS(IP1,JM2)=KT2.OR.(CS(IP1,JM2).AND.MKC(10))
      GO TO 818
  806 IF (ML+MB.NE.4) GO TO 807
      JP2=J+2
      IP2=I+2
    8 KT2=2*IMP(10)
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      CS(IP2,JP2)=KT2.OR.(CS(IP2,JP2).AND.MKC(10))
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      GO TO 818
  807 IF (ML+MT.NE.4) GO TO 808
      JM2=J-2
      IP2=I+2
    9 KT2=2*IMP(10)
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      CS(IP2,JM2)=KT2.OR.(CS(IP2,JM2).AND.MKC(10))
      CS(IP1,JM2)=KT2.OR.(CS(IP1,JM2).AND.MKC(10))
      CS(I,JM2)=KT2.OR.(CS(I,JM2).AND.MKC(10))
      CS(IM1,JM2)=KT2.OR.(CS(IM1,JM2).AND.MKC(10))
      GO TO 818
  808 IF (MQ+MR.NE.4) GO TO 809
      JP2=J+2
      IM2=I-2
   10 KT2=2*IMP(10)
      CS(IM2,JM1)=KT2.OR.(CS(IM2,JM1).AND.MKC(10))
      CS(IM2,J)=KT2.OR.(CS(IM2,J).AND.MKC(10))
      CS(IM2,JP2)=KT2.OR.(CS(IM2,JP2).AND.MKC(10))
      CS(IM2,JP1)=KT2.OR.(CS(IM2,JP1).AND.MKC(10))
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      GO TO 818
  809 IF (MR+MT.NE.4) GO TO 810
      JM2=J-2
      IM2=I-2
   11 KT2=2*IMP(10)
      CS(IP1,JM2)=KT2.OR.(CS(IP1,JM2).AND.MKC(10))
      CS(I,JM2)=KT2.OR.(CS(I,JM2).AND.MKC(10))
      CS(IM1,JM2)=KT2.OR.(CS(IM1,JM2).AND.MKC(10))
      CS(IM2,JM2)=KT2.OR.(CS(IM2,JM2).AND.MKC(10))
      CS(IM2,JM1)=KT2.OR.(CS(IM2,JM1).AND.MKC(10))
      CS(IM2,J)=KT2.OR.(CS(IM2,J).AND.MKC(10))
      CS(IM2,JP1)=KT2.OR.(CS(IM2,JP1).AND.MKC(10))
      GO TO 818
  810 IF (MQ+MT.NE.4) GO TO 811
      IM2=I-2
      IP2=I+2
   12 KT2=2*IMP(10)
      CS(IM2,JP1)=KT2.OR.(CS(IM2,JP1).AND.MKC(10))
      CS(IM2,J)=KT2.OR.(CS(IM2,J).AND.MKC(10))
      CS(IM2,JM1)=KT2.OR.(CS(IM2,JM1).AND.MKC(10))
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      GO TO 818
  811 IF (ML.NE.2) GO TO 812
      JM2=J-2
      JP2=J+2
      IP2=I+2
   13 KT2=2*IMP(10)
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      CS(IP2,JP2)=KT2.OR.(CS(IP2,JP2).AND.MKC(10))
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      CS(IP2,JM2)=KT2.OR.(CS(IP2,JM2).AND.MKC(10))
      CS(IP1,JM2)=KT2.OR.(CS(IP1,JM2).AND.MKC(10))
      CS(I,JM2)=KT2.OR.(CS(I,JM2).AND.MKC(10))
      CS(IM1,JM2)=KT2.OR.(CS(IM1,JM2).AND.MKC(10))
      GO TO 818
  812 IF (MR.NE.2) GO TO 813
      JM2=J-2
      JP2=J+2
      IM2=I-2
   14 KT2=2*IMP(10)
      CS(IP1,JM2)=KT2.OR.(CS(IP1,JM2).AND.MKC(10))
      CS(I,JM2)=KT2.OR.(CS(I,JM2).AND.MKC(10))
      CS(IM1,JM2)=KT2.OR.(CS(IM1,JM2).AND.MKC(10))
      CS(IM2,JM2)=KT2.OR.(CS(IM2,JM2).AND.MKC(10))
      CS(IM2,JM1)=KT2.OR.(CS(IM2,JM1).AND.MKC(10))
      CS(IM2,J)=KT2.OR.(CS(IM2,J).AND.MKC(10))
      CS(IM2,JP1)=KT2.OR.(CS(IM2,JP1).AND.MKC(10))
      CS(IM2,JP2)=KT2.OR.(CS(IM2,JP2).AND.MKC(10))
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      GO TO 818
  813 IF (MQ.NE.2) GO TO 814
      JP2=J+2
      IM2=I-2
      IP2=I+2
   15 KT2=2*IMP(10)
      CS(IM2,JM1)=KT2.OR.(CS(IM2,JM1).AND.MKC(10))
      CS(IM2,J)=KT2.OR.(CS(IM2,J).AND.MKC(10))
      CS(IM2,JP1)=KT2.OR.(CS(IM2,JP1).AND.MKC(10))
      CS(IM2,JP2)=KT2.OR.(CS(IM2,JP2).AND.MKC(10))
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      CS(IP2,JP2)=KT2.OR.(CS(IP2,JP2).AND.MKC(10))
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      GO TO 818
  814 IF (MT.NE.2) GO TO 815
      JM2=J-2
      IM2=I-2
      IP2=I+2
   16 KT2=2*IMP(10)
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      CS(IP2,JM2)=KT2.OR.(CS(IP2,JM2).AND.MKC(10))
      CS(IP1,JM2)=KT2.OR.(CS(IP1,JM2).AND.MKC(10))
      CS(I,JM2)=KT2.OR.(CS(I,JM2).AND.MKC(10))
      CS(IM1,JM2)=KT2.OR.(CS(IM1,JM2).AND.MKC(10))
      CS(IM2,JM2)=KT2.OR.(CS(IM2,JM2).AND.MKC(10))
      CS(IM2,JM1)=KT2.OR.(CS(IM2,JM1).AND.MKC(10))
      CS(IM2,J)=KT2.OR.(CS(IM2,J).AND.MKC(10))
      CS(IM2,JP1)=KT2.OR.(CS(IM2,JP1).AND.MKC(10))
      GO TO 818
  815 K9A=CS(I,JP1).AND.MSK(9)
      K9A=K9A*DV(9)
      IF (K9.EQ.K9A) GO TO 816
      IF (K9A.EQ.4) GO TO 816
      K1A=CS(I,JP1).AND.MSK
      IF (K1A.EQ.2) GO TO 818
      JP2=J+2
   17 KT1=2*IMP(11)
      CS(I,J)=KT1.OR.(CS(I,J).AND.MKC(11))
      CS(I,JP1)=KT1.OR.(CS(I,JP1).AND.MKC(11))
      KT2=2*IMP(10)
      CS(I,J)=KT2.OR.(CS(I,J).AND.MKC(10))
      CS(I,JP1)=KT2.OR.(CS(I,JP1).AND.MKC(10))
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      CS(IP1,JP1)=KT2.OR.(CS(IP1,JP1).AND.MKC(10))
      CS(IP1,J)=KT2.OR.(CS(IP1,J).AND.MKC(10))
      CS(IP1,JM1)=KT2.OR.(CS(IP1,JM1).AND.MKC(10))
      CS(I,JM1)=KT2.OR.(CS(I,JM1).AND.MKC(10))
      CS(IM1,J)=KT2.OR.(CS(IM1,J).AND.MKC(10))
      CS(IM1,JM1)=KT2.OR.(CS(IM1,JM1).AND.MKC(10))
      CS(IM1,JP1)=KT2.OR.(CS(IM1,JP1).AND.MKC(10))
      CS(IM1,JP2)=KT2.OR.(CS(IM1,JP2).AND.MKC(10))
  816 K9B=CS(IP1,JP1).AND.MSK(9)
      K9B=K9B*DV(9)
      IF (K9.EQ.K9B) GO TO 817
      IF (K9B.EQ.4) GO TO 817
      K1B=CS(IP1,JP1).AND.MSK
      IF (K1B.EQ.1.OR.K1B.EQ.2) GO TO 818
      JP2=J+2
      IP2=I+2
   18 KT1=2*IMP(11)
      CS(I,J)=KT1.OR.(CS(I,J).AND.MKC(11))
      CS(IP1,JP1)=KT1.OR.(CS(IP1,JP1).AND.MKC(11))
      KT2=2*IMP(10)
      CS(I,J)=KT2.OR.(CS(I,J).AND.MKC(10))
      CS(IP1,JP1)=KT2.OR.(CS(IP1,JP1).AND.MKC(10))
      CS(IP1,JP2)=KT2.OR.(CS(IP1,JP2).AND.MKC(10))
      CS(IP2,JP2)=KT2.OR.(CS(IP2,JP2).AND.MKC(10))
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP1,J)=KT2.OR.(CS(IP1,J).AND.MKC(10))
      CS(IP1,JM1)=KT2.OR.(CS(IP1,JM1).AND.MKC(10))
      CS(I,JM1)=KT2.OR.(CS(I,JM1).AND.MKC(10))
      CS(IM1,JM1)=KT2.OR.(CS(IM1,JM1).AND.MKC(10))
      CS(IM1,J)=KT2.OR.(CS(IM1,J).AND.MKC(10))
      CS(IM1,JP1)=KT2.OR.(CS(IM1,JP1).AND.MKC(10))
      CS(I,JP1)=KT2.OR.(CS(I,JP1).AND.MKC(10))
      CS(I,JP2)=KT2.OR.(CS(I,JP2).AND.MKC(10))
  817 K9C=CS(IP1,J).AND.MSK(9)
      K9C=K9C*DV(9)
      IF (K9.EQ.K9C) GO TO 818
      IF (K9C.EQ.4) GO TO 818
      K1C=CS(IP1,J).AND.MSK
      IF (K1C.EQ.2) GO TO 818
      IP2=I+2
   19 KT1=2*IMP(11)
      CS(I,J)=KT1.OR.(CS(I,J).AND.MKC(11))
      CS(IP1,J)=KT1.OR.(CS(IP1,J).AND.MKC(11))
      KT2=2*IMP(10)
      CS(I,J)=KT2.OR.(CS(I,J).AND.MKC(10))
      CS(IP1,J)=KT2.OR.(CS(IP1,J).AND.MKC(10))
      CS(IP2,J)=KT2.OR.(CS(IP2,J).AND.MKC(10))
      CS(IP2,JM1)=KT2.OR.(CS(IP2,JM1).AND.MKC(10))
      CS(IP1,JM1)=KT2.OR.(CS(IP1,JM1).AND.MKC(10))
      CS(I,JM1)=KT2.OR.(CS(I,JM1).AND.MKC(10))
      CS(IM1,JM1)=KT2.OR.(CS(IM1,JM1).AND.MKC(10))
      CS(IM1,J)=KT2.OR.(CS(IM1,J).AND.MKC(10))
      CS(IM1,JP1)=KT2.OR.(CS(IM1,JP1).AND.MKC(10))
      CS(I,JP1)=KT2.OR.(CS(I,JP1).AND.MKC(10))
      CS(IP1,JP1)=KT2.OR.(CS(IP1,JP1).AND.MKC(10))
      CS(IP2,JP1)=KT2.OR.(CS(IP2,JP1).AND.MKC(10))
  818 CONTINUE
      RETURN
      END
      SUBROUTINE BNDCND(KKKK)
      COMMON/TVPCOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,TWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MC,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVC,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNRP,NRP,NRP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVF,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      CALL FSRDCD(KKKK)
      DO 517 J=1,NJ
      DO 517 I=1,NI
      K1=CS(I,J).AND.MSK
      IF (K1.NE.2) GO TO 517
      K7=CS(I,J).AND.MSK(7)
      K7=K7*DV(7)
      K5=CS(I,J).AND.MSK(5)
      K5=K5*DV(5)
      K2=CS(I,J).AND.MSK(2)
      K2=K2*DV(2)
      IF (K5.EQ.2) GO TO 400
      S=1.
      IF (K7.EQ.4) GO TO 509
      IF (K7.EQ.3) GO TO 508
      IF (K7.EQ.2) GO TO 500
      I1=I+1
      I2=I1
      I3=I
      I4=I+2
      GO TO 501
  500 I1=I
      I2=I-1
      I3=I+1
      I4=I2
  501 GO TO (502,507,504,505,517),K2
  502 IF (IVC.NE.0) GO TO 507
      S=-1.
      GO TO 507
  504 SS=-1.
      GO TO 506
  505 S=-1.
      SS=1.
  506 UT(I3,J)=SS*UT(I4,J)
      UT(I1,J)=0.
  507 VT(I,J+1)=S*VT(I2,J+1)
      VT(I,J)=S*VT(I2,J)
      GO TO 517
  508 J1=J+1
      J2=J1
      J3=J
      J4=J+2
      GO TO 510
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509 J1=J
    J2=J-1
    J3=J+1
    J4=J2
510 GO TO (511,516,513,514,517),K2
511 IF (IVO.NE.0) GO TO 516
    S=-1.
    GO TO 516
513 SS=-1.
    GO TO 515
514 S=-1.
    SS=1.
515 VT(I,J3)=SS*VT(I,J4)
    VT(I,J1)=0.
516 UT(I+1,J)=S*UT(I+1,J2)
    UT(I,J)=S*UT(I,J2)
    GO TO 517
400 K4=CS(I,J-1).AND.MSK(4)
    K4=K4*DV(4)
    IF (K2.EQ.3) GO TO 401
    S=-1.
    GO TO 402
401 S=1.
402 IF (K4.EQ.2) GO TO 404
    IF (K7.EQ.1) GO TO 403
    UT(I,J)=0.
    VT(I,J+1)=0.
    VT(I,J)=S*VT(I-1,J)
    UT(I+1,J)=S*UT(I+1,J+1)
    GO TO 517
403 UT(I+1,J)=0.
    VT(I,J+1)=0.
    VT(I,J)=S*VT(I+1,J)
    UT(I,J)=S*UT(I,J+1)
    GO TO 517
404 IF (K7.EQ.1) GO TO 405
    UT(I,J)=0.
    VT(I,J)=0.
    VT(I,J+1)=S*VT(I-1,J+1)
    UT(I+1,J)=S*UT(I+1,J-1)
    GO TO 517
405 UT(I+1,J)=0.
    VT(I,J)=0.
    VT(I,J+1)=S*VT(I+1,J+1)
    UT(I,J)=S*UT(I,J-1)
517 CONTINUE
    CALL OTBDCD
    IF (IVO.NE.0) CALL INBDCD
    IF (KKKK.EQ.2) GO TO 521
    DO 519 J=1,NJ
    U(NIP1,J)=UT(NIP1,J)
    DO 519 I=1,NI
    U(I,J)=UT(I,J)
519 V(I,J)=VT(I,J)
    DO 520 I=1,NI
520 V(I,NJP1)=VT(I,NJP1)
521 RETURN
    END
    SUBROUTINE OTBDCD
    COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
    COMMON/TVGUIDE/TMODE,TEXT,ITV
    COMMON/TVFACT/FACT
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    COMMON/TVTUNE/LPEN,LPEF,ITAL,TWINK,INTS,IPT,IUP
    COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
   $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
    COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
   $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
   $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
   $       V0,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
    COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
   $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
   $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
   $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
   $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
   $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
    COMMON KNBP,NHP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
   $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
    REAL MU,MU1,MU2,NK,NKT
    INTEGER CS,PS
    DO 603 J=1,NJ
    DO 603 I=1,NI
    K2=CS(I,J).AND.MSK(2)
    K2=K2*DV(2)
    IF (K2.NE.2) GO TO 603
    K7=CS(I,J).AND.MSK(7)
    K7=K7*DV(7)
    IF (K7.EQ.4) GO TO 602
    IF (K7.EQ.3) GO TO 601
    IF (K7.EQ.2) GO TO 600
    UT(I+1,J)=UT(I+2,J)+DXC*ODY*(VT(I+1,J+1)-VT(I+1,J))
    UT(I,J)=UT(I+1,J)+DXC*ODY*(VT(I,J+1)-VT(I,J))
    GO TO 603
600 UT(I,J)=UT(I-1,J)+DXC*ODY*(VT(I-1,J)-VT(I-1,J+1))
    UT(I+1,J)=UT(I,J)+DXC*ODY*(VT(I,J)-VT(I,J+1))
    GO TO 603
601 VT(I,J+1)=VT(I,J+2)+DYC*ODX*(UT(I+1,J+1)-UT(I,J+1))
    VT(I,J)=VT(I,J+1)+DYC*ODX*(UT(I+1,J)-UT(I,J))
    GO TO 603
602 VT(I,J)=VT(I,J-1)+DYC*ODX*(UT(I,J-1)-UT(I+1,J-1))
    VT(I,J+1)=VT(I,J)+DYC*ODX*(UT(I,J)-UT(I+1,J))
603 CONTINUE
    RETURN
    END
    SUBROUTINE INBDCD
    COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
    COMMON/TVGUIDE/TMODE,TEXT,ITV
    COMMON/TVFACT/FACT
    COMMON/TVTUNE/LPEN,LPEF,ITAL,TWINK,INTS,IRT,IUP
    COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
   $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
    COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
   $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
   $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
   $       V0,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
    COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
   $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
   $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
   $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
   $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
   $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
    COMMON KNBP,NHP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
   $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
    REAL MU,MU1,MU2,NK,NKT
    INTEGER CS,PS
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      DO 703 J=1,NJ
      DO 703 I=1,NI
      K2=CS(I,J).AND.MSK(2)
      K2=K2*DV(2)
      IF (K2.NE.1) GO TO 703
      K7=CS(I,J).AND.MSK(7)
      K7=K7*DV(7)
      IF (K7.EQ.4) GO TO 702
      IF (K7.EQ.3) GO TO 701
      IF (K7.EQ.2) GO TO 700
      UT(I,J)=UT(I+2,J)
      UT(I+1,J)=UT(I,J)
      GO TO 703
  700 UT(I,J)=UT(I-1,J)
      UT(I+1,J)=UT(I,J)
      GO TO 703
  701 VT(I,J)=VT(I,J+2)
      VT(I,J+1)=VT(I,J)
      GO TO 703
  702 VT(I,J)=VT(I,J-1)
      VT(I,J+1)=VT(I,J)
  703 CONTINUE
      RETURN
      END
      SUBROUTINE FSBDCD(KKKK)
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/IMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPFN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
     $       V0,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVF,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      IF (KKKK.EQ.3) GO TO 805
      DO 101 J=2,NJM1
      DO 101 I=2,NIM1
      K1=CS(I,J).AND.MSK
      IF (K1.NE.5) GO TO 101
      K1B=CS(I+1,J).AND.MSK
      IF (K1B.NE.3) GO TO 100
      UT(I+1,J)=U(I+1,J)+GXDT
      GO TO 101
  100 K1D=CS(I,J+1).AND.MSK
      IF (K1D.NE.3) GO TO 101
      VT(I,J+1)=V(I,J+1)+GYDT
  101 CONTINUE
      DO 411 J=2,NJM1
      DO 411 I=2,NIM1
      K1=CS(I,J).AND.MSK
      IF (K1.NE.5) GO TO 411
      K1A=CS(I-1,J).AND.MSK
      IF (K1A.NE.3) GO TO 407
      UT(I,J)=UT(I+1,J)
      GO TO 408
  407 K1B=CS(I+1,J).AND.MSK
      IF (K1B.NE.3) GO TO 408
      UT(I+1,J)=UT(I,J)
  408 K1C=CS(I,J-1).AND.MSK
      IF (K1C.NE.3) GO TO 409
      VT(I,J)=VT(I,J+1)
  409 K1D=CS(I,J+1).AND.MSK
      GO TO 411
      IF (K1D.NE.3) GO TO 411
      VT(I,J+1)=VT(I,J)
  411 CONTINUE
      DO 415 J=2,NJM1
      JM=J-1
      JP=J+1
      DO 415 I=2,NIM1
      IM=I-1
      IP=I+1
      K1=CS(I,J).AND.MSK
      IF (K1.NE.5) GO TO 415
      K1A=CS(IM,J).AND.MSK
      K1B=CS(IP,J).AND.MSK
      K1C=CS(I,JM).AND.MSK
      K1D=CS(I,JP).AND.MSK
      IF (K1A.NE.3.AND.K1D.EQ.3.AND.K1B.EQ.3.AND.(K1C.EQ.4.OR.K1C.EQ.5))
     $   GO TO 416
      IF (K1A.EQ.3.AND.K1D.EQ.3.AND.K1B.NE.3.AND.(K1C.EQ.4.OR.K1C.EQ.5))
     $   GO TO 417
      IF (K1A.NE.3.AND.(K1D.EQ.4.OR.K1D.EQ.5).AND.K1B.EQ.3.AND.K1C.EQ.3)
     $   GO TO 418
      IF (K1A.EQ.3.AND.(K1D.EQ.4.OR.K1D.EQ.5).AND.K1B.NE.3.AND.K1C.EQ.3)
     $   GO TO 419
      IF (K1A.EQ.3.AND.K1D.EQ.3.AND.K1B.EQ.3.AND.(K1C.EQ.4.OR.K1C.EQ.5))
     $ GO TO 420
      IF ((K1A.EQ.4.OR.K1A.EQ.5).AND.K1D.EQ.3.AND.K1B.EQ.3.AND.K1C.EQ.3)
     $ GO TO 412
      IF (K1A.EQ.3.AND.(K1D.EQ.4.OR.K1D.EQ.5).AND.K1B.EQ.3.AND.K1C.EQ.3)
     $ GO TO 413
      IF (K1A.EQ.3.AND.K1D.EQ.3.AND.(K1B.EQ.4.OR.K1B.EQ.5).AND.K1C.EQ.3)
     $ GO TO 414
      GO TO 415
  416 UT(IP,J)=UT(IP,JM)+UT(IP,JM)-UT(IP,J-2)
      VT(I,JP)=VT(I,J)+VT(I,J)-VT(I,JM)
      GO TO 415
  417 UT(I,J)=UT(I,JM)+UT(I,JM)-UT(I,J-2)
      VT(I,JP)=VT(I,J)+VT(I,J)-VT(I,JM)
      GO TO 415
  418 UT(IP,J)=UT(IP,JM)+UT(IP,JM)-UT(IP,J-2)
      VT(I,J)=VT(I,JP)+VT(I,JP)-VT(I,J+2)
      GO TO 415
  419 UT(I,J)=UT(I,JM)+UT(I,JM)-UT(I,J-2)
      VT(I,J)=VT(I,JP)+VT(I,JP)-VT(I,J+2)
      GO TO 415
  420 VT(I,JP)=VT(I,J)+VT(I,J)-VT(I,JM)
      GO TO 415
  412 UT(IP,J)=UT(I,J)+UT(I,J)-UT(IM,J)
      GO TO 415
  413 VT(I,J)=VT(I,JP)+VT(I,JP)-VT(I,J+2)
      GO TO 415
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414 UT(I,J)=UT(IP,J)+UT(IP,J)-UT(I+2,J)
415 CONTINUE
    DO 804 J=2,NJM1
    JM=J-1
    JP=J+1
    DO 804 I=2,NIM1
    K1=CS(I,J).AND.MSK
    IF (K1.NE.5) GO TO 804
    IM=I-1
    IP=I+1
    K1A=CS(IM,J).AND.MSK
    K1B=CS(IP,J).AND.MSK
    K1C=CS(I,JM).AND.MSK
    K1D=CS(I,JP).AND.MSK
    IF(K1C.EQ.3.AND.K1A.NE.3.AND.K1B.NE.3
   $    .AND.(K1D.EQ.4.OR.K1D.EQ.5)) GO TO 800
    IF(K1D.EQ.3.AND.K1A.NE.3.AND.K1B.NE.3
   $    .AND.(K1C.EQ.4.OR.K1C.EQ.5)) GO TO 801
    IF (K1A.EQ.3.AND.(K1B.EQ.4.OR.K1B.EQ.5).AND.K1C.NE.3.AND.K1D.NE.3)
   $ GO TO 802
    IF (K1B.EQ.3.AND.(K1A.EQ.4.OR.K1A.EQ.5).AND.K1C.NE.3.AND.K1D.NE.3)
   $ GO TO 803
    GO TO 804
800 VT(I,J)=VT(I,JP)+VT(I,JP)-VT(I,J+2)
    GO TO 804
801 VT(I,JP)=VT(I,J)+VT(I,J)-VT(I,JM)
    GO TO 804
802 UT(I,J)=UT(IP,J)+UT(IP,J)-UT(I+2,J)
    GO TO 804
803 UT(IP,J)=UT(I,J)+UT(I,J)-UT(IM,J)
804 CONTINUE
805 DO 806 J=2,NJM1
    JM=J-1
    JP=J+1
    DO 806 I=2,NIM1
    K1=CS(I,J).AND.MSK
    IF (K1.NE.5) GO TO 806
    IM=I-1
    IP=I+1
    K1A=CS(IM,J).AND.MSK
    K1B=CS(IP,J).AND.MSK
    K1C=CS(I,JM).AND.MSK
    K1D=CS(I,JP).AND.MSK
    K1E=CS(IM,JP).AND.MSK
    K1F=CS(IP,JP).AND.MSK
    K1G=CS(IP,JM).AND.MSK
    K1H=CS(IM,JM).AND.MSK
    IF (K1B.NE.3.OR.K1F.NE.3) GO TO 807
    VT(IP,JP)=VT(I,JP)
    IF (K1A.EQ.4.OR.K1A.EQ.5) VT(IP,JP)=VT(I,JP)+VT(I,JP)-VT(IM,JP)
807 IF (K1A.NE.3.OR.K1E.NE.3) GO TO 808
    VT(IM,JP)=VT(I,JP)
    IF (K1B.EQ.4.OR.K1B.EQ.5) VT(IM,JP)=VT(I,JP)+VT(I,JP)-VT(IP,JP)
808 IF (K1D.NE.3.OR.K1F.NE.3) GO TO 809
    UT(IP,JP)=UT(IP,J)
    IF (K1C.EQ.4.OR.K1C.EQ.5) UT(IP,JP)=UT(IP,J)+UT(IP,J)-UT(IP,JM)
809 IF (K1C.NE.3.OR.K1G.NE.3) GO TO 810
    UT(IP,JM)=UT(IP,J)
    IF (K1D.EQ.4.OR.K1D.EQ.5) UT(IP,JM)=UT(IP,J)+UT(IP,J)-UT(IP,JP)
810 IF (K1B.NE.3.OR.K1G.NE.3) GO TO 811
    VT(IP,J)=VT(I,J)
    IF (K1A.EQ.4.OR.K1A.EQ.5) VT(IP,J)=VT(I,J)+VT(I,J)-VT(IM,J)
811 IF (K1A.NE.3.OR.K1H.NE.3) GO TO 812
    VT(IM,J)=VT(I,J)
    IF (K1B.EQ.4.OR.K1B.EQ.5) VT(IM,J)=VT(I,J)+VT(I,J)-VT(IP,J)
812 IF (K1D.NE.3.OR.K1E.NE.3) GO TO 813
    UT(I,JP)=UT(I,J)
    IF (K1C.EQ.4.OR.K1C.EQ.5) UT(I,JP)=UT(I,J)+UT(I,J)-UT(I,JM)
813 IF (K1C.NE.3.OR.K1H.NE.3) GO TO 806
    UT(I,JM)=UT(I,J)
    IF (K1D.EQ.4.OR.K1D.EQ.5) UT(I,JM)=UT(I,J)+UT(I,J)-UT(I,JP)
806 CONTINUE
    RETURN
    END
    SUBROUTINE CNTROL
    COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
    COMMON/TVGUIDE/TMODE,TEXT,ITV
    COMMON/TVFACT/FACT
    COMMON/TVTUNE/LPEN,LREF,ITAL,IWINK,INTS,IRT,IUP
    COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
   $       DTPP,DTVP,DXC,DXCD2,DX14,DXIN,DXP,DYC,DYCD2,DY14,DYIN,DYP
    COMMON EPS,G,GM,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
   $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
   $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
   $       V0,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
    COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
   $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
   $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
   $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
   $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
   $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
    COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
   $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,NUM(5)
    REAL MU,MU1,MU2,NK,NKT
    INTEGER CS,PS
    DIMENSION AA(31107),JTIME(7)
    EQUIVALENCE (AA,DELTAS)
    CALL STATUS (JTIME)
    IF (ISAVE.GT.0) GO TO 607
    IF (T.NE.0.) GO TO 600
    CALL PLTPAR
    CALL CELPRT
    TP=DTP
    TCP=DTCP
    TPP=DTPP
    TVP=DTVP
    TPR=DTPR
    WRITE (NTP) DUMMY
    GO TO 605
600 IF (KD.EQ.2) GO TO 606
    IF (JTIME(3).GT.15000) GO TO 595
    CALL PLTPAR
    CALL PRSPLT
    GO TO 596
595 STIM=JTIME(1)
    ITIME=STIM*.001
    CALL PRSPLT
    IF (T.LT.TP-.00000001) GO TO 601
    CALL PLTPAR
    TP=T+DTP
601 IF(T.LT.TCP-.00000001) GO TO 602
    CALL CELPRT
596 WRITE (MO,6002) T,ITIME
    WRITE (NTP) AA
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      IF (JTIME(3).LE.15000) GO TO 608
      TCP=T+DTCP
  602 IF (T.LT.TPP-.0000001) GO TO 603
      CALL PAPPRT
      TPP=T+DTPP
  603 IF (T.LT.TVP-.0000001) GO TO 604
      CALL PLTVEL
      TVP=T+DTVP
  604 IF (T.GE.TL-.0000001) KD=2
  605 T=T+DT
      WRITE (MO,6004) JTIME(1)
      RETURN
  606 CALL CELPRT
      CALL PARPRT
      WRITE (MO,6000)
      GO TO 605
  607 DO 609 IDUM=1,IWDG
 609  READ(NTP) DUMMY
      READ(NTP) AA
      IF(MOD(IFIX(T+.5),5 ).NE.0) BACKSPACE NTP
      PLMIN=XI
      PLMAX=XA
      XMIN=XI
      XMAX=XA
      ISAVE=0
      WRITE (MO,6003) T
      GO TO 605
  608 KD=2
      END FILE NTP
      REWIND NTP
      GO TO 605
 6000 FORMAT(1H1,10X,41HABNORMAL STOP -- LOOK FOR ANOTHER MESSAGE)
 6001 FORMAT(7F10.0)
 6002 FORMAT(1H-,10X,15HSAVING AT  T = ,F6.3,25H  --  ACCUMULATED TIME =
     * ,I6,8H SECONDS)
 6003 FORMAT(1H1,10X,18HRESTARTING AT T = ,F6.3/1H1)
 6004 FORMAT(1H0,30X,24H*****  ELAPSED TIME  =  ,I8,21H  MILLISECONDS  *
     ******//)
      END
      SUBROUTINE PLTPAR
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,TWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     *       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GM,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     *       MT,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     *       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
     *       V0,W,IVC,DXODY,DYODX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     *       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     *       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     *       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     *       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     *       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     *       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DIMENSION X1(3000),Y1(3000)
      CALL PLTBND
      TEXT=1.
      KK1=0
      KK2=0
      KK3=0
      DO 702 K=1,NP
      KP=PS(K).AND.MSK
      IF (KP.EQ.3) GO TO 702
      IF (IWDG.LE.0) GO TO 604
      X=(5.*XP(K)-YP(K))/5.2
      Y=(YP(K)+.2*XP(K))/1.04
      GO TO 605
  604 X=XP(K)
      Y=YP(K)
  605 I=XP(K)*ODX+2.
      J=YP(K)*ODY+2.
      K1=CS(I,J).AND.MSK
      IF (K1.EQ.2) GO TO 702
      IF (K1.EQ.1) GO TO 702
      KK1=KK1+1
      X1(KK1)=X
      Y1(KK1)=Y
  702 CONTINUE
      IF (KK1.EQ.0) GO TO 701
      CALL TVPLOT(X1,Y1,KK1)
  701 DO 703 K=1,NP
      KP=PS(K).AND.MSK
      IF (KP.EQ.3) GO TO 703
      KPA=PS(K).AND.MSK(3)
      KPA=KPA*DV(3)
      IF (KPA.NE.2) GO TO 703
      IF (IWDG.LE.0) GO TO 606
      X=(5.*XP(K)-YP(K))/5.2
      Y=(YP(K)+.2*XP(K))/1.04
      GO TO 607
  606 X=XP(K)
      Y=YP(K)
  607 I=XP(K)*ODX+2.
      J=YP(K)*ODY+2.
      K1=CS(I,J).AND.MSK
      IF (K1.EQ.1.OR.K1.EQ.2) GO TO 703
      KK2=KK2+1
      X1(KK2)=X
      Y1(KK2)=Y
  703 CONTINUE
      IF (KK2.EQ.0) GO TO 900
      DO 704 L=1,5
  704 CALL TVPLOT(X1,Y1,KK2)
  900 DO 903 K=1,NP
      KP=PS(K).AND.MSK
      IF (KP.EQ.3) GO TO 903
      KPB=PS(K).AND.MSK(5)
      KPB=KPB*DV(5)
      IF (KPB.EQ.1) GO TO 903
      IF (IWDG.LE.0) GO TO 901
      X=(5.*XP(K)-YP(K))/5.2
      Y=(YP(K)+2.*XP(K))/1.04
      GO TO 902
  901 X=XP(K)
      Y=YP(K)
  902 I=XP(K)*ODX+2.
      J=YP(K)*ODY+2.
      K1=CS(I,J).AND.MSK
      IF (K1.EQ.1.OR.K1.EQ.2) GO TO 903
      KK3=KK3+1
      X1(KK3)=X
      Y1(KK3)=Y
  903 CONTINUE
      IF (KK3.EQ.0) GO TO 705
      DO 904 L=1,10
  904 CALL TVPLOT(X1,Y1,KK3)
  705 CALL TVNEXT
      RETURN
      END
      SUBROUTINE PLTBND
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DIMENSION X1(21),Y1(21)
      TEXT=0.
      IF (KNBP.EQ.2) GO TO 601
      DO 600 M=1,NBP
      IF (XX1(M).NE.0) GO TO 600
      X1(1)=XB1(M)
      X1(2)=XB1(M+1)
      Y1(1)=YB1(M)
      Y1(2)=YB1(M+1)
      DO 598 I=1,2
  598 CALL TVPLOT(X1,Y1,2)
  600 CONTINUE
      IF (KNBP.EQ.1) GO TO 603
  601 DO 602 M=1,NBP2
      IF (XX2(M).NE.0) GO TO 602
      X1(1)=XB2(M)
      X1(2)=XB2(M+1)
      Y1(1)=YB2(M)
      Y1(2)=YB2(M+1)
      DO 599 I=1,2
  599 CALL TVPLOT(X1,Y1,2)
  602 CONTINUE
  603 WRITE (98,6000) T
      DO 606 I=1,2
  606 CALL TVLTR(824.,72.,0,3)
      RETURN
 6000 FORMAT(4HT = ,F6.3)
      END
      SUBROUTINE CELPRT
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
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      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      LINE=0
      WRITE (MO,8000) T
      WRITE (MO,8001)
      DO 800 J=1,NJ
      DO 800 I=1,NI
      IF (LINE.LT.50) GO TO 801
      LINE=0
      WRITE (MO,8003)
      WRITE (MO,8001)
  801 UU=.5*(U(I+1,J)+U(I,J))
      VV=.5*(V(I,J+1)+V(I,J))
      VEL=SQRT(UU*UU+VV*VV)
      FR=P(I,J)/(R(I,J)*VEL*VEL)
      WRITE (MO,8002) I,J,UU,VV,VEL,P(I,J),R(I,J),MU(I,J),FR,CS(I,J)
      LINE=LINE+1
  800 CONTINUE
      WRITE (MO,8004) T
      RETURN
 8000 FORMAT(1H1,10X22HCELL PRINT FOR TIME = ,F6.3///)
 8001 FORMAT(1H ,5X1HI,3X1HJ,10X4HUBAR,10X4HVBAR,4X10HTOTAL VEL.,6X8HPRE
     $SSURE,7X7HDENSITY,5X9HVISCOSITY,4X10H PRES COEF,7X10HCELL FLAGS/)
 8002 FORMAT(1H ,4X,I2,I4,7E14.5,2X,O15)
 8003 FORMAT(1H1)
 8004 FORMAT(1H0,20X36H******  END OF CELL PRINT FOR TIME = ,F6.3,7H  ***
     $**/1H1)
      END
      SUBROUTINE PARPRT
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
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NOT REPRODUCIBLE

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      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      LINE=0
      WRITE (MO,9000) T
      WRITE (MO,9001)
      DO 900 K=1,NP
      KP=PS(K).AND.MSK
      IF (KP.EQ.3) GO TO 900
      WRITE (MO,9002) K,XP(K),YP(K),UP(K),VP(K),PS(K)
      LINE=LINE+1
      IF (LINE.NE.50) GO TO 900
      LINE=0
      WRITE (MO,9003)
      WRITE (MO,9001)
  900 CONTINUE
      WRITE (MO,9004) T
      RETURN
 9000 FORMAT(1H1,10X26HPARTICLE PRINT FOR TIME = ,F6.3///)
 9001 FORMAT(1H ,9X1HK,14X1HX,14X1HY,14X1HU,14X1HV,5X2HPS)
 9002 FORMAT(1H ,I10,4E15.6,2X,O5)
 9003 FORMAT(1H1)
 9004 FORMAT(1H0,20X40H*****  END OF PARTICLE PRINT FOR TIME = ,F6.3,7H
     $ *****/1H1)
      END
      SUBROUTINE PLTVEL
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DY,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DIMENSION XV(2),YV(2)
      CALL PLTBND
      TEXT=0.
      DO 100 J=2,NJM1
      DO 100 I=2,NIM1
      KI=CS(I,J).AND.MSK
      IF (KI.NE.4.AND.KI.NE.5) GO TO 100
      XV(1)=XC(I)
      XV(2)=XV(1)+DELTAS*(U(I+1,J)+U(I,J))
      YV(1)=YC(J)
      YV(2)=YV(1)+DELTAS*(V(I,J+1)+V(I,J))
      DO 99 L=1,2
   99 CALL TVPLOT(XV,YV,2)
  100 CONTINUE
      CALL TVNEXT
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      RETURN
      END
      SUBROUTINE PRSPLT
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,I[illegible]TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DIMENSION PC(50)
      IF (T.LT.TPR-.000001) GO TO 939
      IF (NC.EQ.0) GO TO 939
      TPR=T+DTPR
      CALL PLTBND
      TEXT=0.
      PMIN=0.
      PMAX=0.
      DO 900 J=1,NJ
      DO 900 I=1,NI
      PMIN=AMIN1(PMIN,P(I,J))
  900 PMAX=AMAX1(PMAX,P(I,J))
      DP=(PMAX+PMAX-PMIN-PMIN)/FLOAT(NC-1)
      DP=DP.AND.77774000000000000000B
      IF (PMIN.LT.0.) GO TO 903
      SUM=.5*DP
  901 IF (SUM.GT.PMIN) GO TO 902
      SUM=SUM+DP
      GO TO 901
  902 SUM=SUM-DP
      GO TO 905
  903 SUM=.5*DP
  904 IF (SUM.LT.PMIN) GO TO 905
      SUM=SUM-DP
      GO TO 904
  905 DO 906 L=1,NC
      PC(L)=SUM
  906 SUM=SUM+DP
      LP=NC
      DO 800 L=1,NC
      IF (PC(L).LE.PMAX) GO TO 800
      LP=L
      GO TO 801
  800 CONTINUE
  801 CONTINUE
      WRITE (MO,9000) PC(1),PC(LP),DP,T
      DO 938 J=2,NJM1
      J=J
      JP=J+1
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NOT REPRODUCIBLE

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      DO 938 I=2,NIM1
      I=I
      IP=I+1
      K1=CS(I,J).AND.MSK
      K1A=CS(I,JP).AND.MSK
      K1B=CS(IP,J).AND.MSK
      K1C=CS(IP,JP).AND.MSK
      IF (K1.EQ.2.OR.K1A.EQ.2.OR.K1B.EQ.2.OR.K1C.EQ.2) GO TO 938
      IF (P(I,J)-P(I,JP)) 907,914,908
  907 JH=JP
      JL=J
      GO TO 909
  908 JH=J
      JL=JP
  909 DO 913 L=1,NC
      IF (PC(L).LE.P(I,JL).OR.PC(L).GE.P(I,JH)) GO TO 913
      IF (PC(L)-P(IP,J)) 910,913,911
  910 J4=JL
      GO TO 912
  911 J4=JH
  912 CALL PSTORE(I,JL,I,JH,IP,J,I,J4,PC(L))
  913 CONTINUE
  914 IF (P(I,J)-P(IP,J)) 915,922,916
  915 IH=IP
      IL=I
      GO TO 917
  916 IH=I
      IL=IP
  917 DO 921 L=1,NC
      IF (PC(L).LE.P(IL,J).OR.PC(L).GE.P(IH,J)) GO TO 921
      IF (PC(L)-P(I,JP)) 918,921,919
  918 IF (PC(L).GT.P(IP,J)) 920,921
  919 IF (PC(L).GE.P(IP,J)) GO TO 921
  920 CALL PSTORE(IL,J,IH,J,I,JP,IP,J,PC(L))
  921 CONTINUE
  922 IF (P(IP,J)-P(IP,JP)) 923,930,924
  923 JH=JP
      JL=J
      GO TO 925
  924 JH=J
      JL=JP
  925 DO 929 L=1,NC
      IF (PC(L).LE.P(IP,JL).OR.PC(L).GE.P(IP,JH)) GO TO 929
      IF (PC(L)-P(I,JP)) 926,929,927
  926 J4=JL
      GO TO 928
  927 J4=JH
  928 CALL PSTORE(IP,JL,IP,JH,I,JP,IP,J4,PC(L))
  929 CONTINUE
  930 IF (P(I,JP)-P(IP,JP)) 931,938,932
  931 IH=IP
      IL=I
      GO TO 933
  932 IH=I
      IL=IP
  933 DO 937 L=1,NC
      IF (PC(L).LE.P(IL,JP).OR.PC(L).GE.P(IH,JP)) GO TO 937
      IF (PC(L)-P(IP,J)) 934,937,935
  934 IF (PC(L).GT.P(I,JP)) 936,937
  935 IF (PC(L).GE.P(I,JP)) GO TO 937
  936 CALL PSTORE(IL,JP,IH,JP,IP,J,I,JP,PC(L))
  937 CONTINUE
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  938 CONTINUE
      CALL TVNEXT
  939 RETURN
 9000 FORMAT(1H ,10X7HPMIN = ,E12.5,5X7HPMAX = ,E12.5,5X7HDELP = ,E12.5,
     $5X7HTIME = ,F6.3)
      END
      SUBROUTINE PSTORE(IL,JL,IH,JH,I3,J3,I4,J4,PPC)
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVC,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWNG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DIMENSION XXC(2),YYC(2)
      T1=(P(IH,JH)-PPC)/(P(IH,JH)-P(IL,JL))
      T2=(P(I4,J4)-PPC)/(P(I4,J4)-P(I3,J3))
      XXC(1)=XC(IH)-T1*(XC(IH)-XC(IL))
      YYC(1)=YC(JH)-T1*(YC(JH)-YC(JL))
      XXC(2)=XC(I4)-T2*(XC(I4)-XC(I3))
      YYC(2)=YC(J4)-T2*(YC(J4)-YC(J3))
      CALL TVPLOT(XXC,YYC,2)
      RETURN
      END
      SUBROUTINE ONTPRS(KKKK)
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVC,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWNG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      IF (KKKK.EQ.2) GO TO 525
      DO 520 J=1,NJM1
      JM=J-1
      JP=J+1
      DO 520 I=1,NIM1
      IM=I-1
      IP=I+1
      IIII=1
      JJJJ=1
      UM1=U(IP,JM)
      UP1=U(IP,JP)
      VM1=V(IM,JP)
      VP1=V(IP,JP)
      K1=CS(I,J).AND.MSK
      IF (K1.NE.2) GO TO 501
      K1B=CS(IP,J).AND.MSK
      IF (K1B.EQ.4) GO TO 500
      IF (K1B.NE.5) IIII=2
  500 K1D=CS(I,JP).AND.MSK
      IF (K1D.EQ.4) GO TO 510
      IF (K1D.NE.5) JJJJ=2
      GO TO 510
  501 IF (K1.EQ.4) GO TO 502
      IF (K1.NE.5) GO TO 520
  502 K1B=CS(IP,J).AND.MSK
      IF (K1B.EQ.3) IIII=2
      K1D=CS(I,JP).AND.MSK
      IF (K1D.EQ.3) JJJJ=2
      IF (IIII.EQ.2) GO TO 506
      K2C=CS(I,JM).AND.MSK(2)
      K2C=K2C*DV(2)
      K2D=CS(I,JP).AND.MSK(2)
      K2D=K2D*DV(2)
      K2F=CS(IP,JM).AND.MSK(2)
      K2F=K2F*DV(2)
      K2G=CS(IP,JP).AND.MSK(2)
      K2G=K2G*DV(2)
      K5C=CS(I,JM).AND.MSK(5)
      K5C=K5C*DV(5)
      K5D=CS(I,JP).AND.MSK(5)
      K5D=K5D*DV(5)
      K5F=CS(IP,JM).AND.MSK(5)
      K5F=K5F*DV(5)
      K5G=CS(IP,JP).AND.MSK(5)
      K5G=K5G*DV(5)
      IF((K2D.EQ.4.AND.K5D.EQ.2).OR.(K2G.EQ.4.AND.K5G.EQ.2)) GO TO 503
      IF((K2D.EQ.3.AND.K5D.EQ.2).OR.(K2G.EQ.3.AND.K5G.EQ.2)) UP1=U(IP,J)
      GO TO 504
  503 UP1=-U(IP,J)
  504 IF((K2C.EQ.4.AND.K5C.EQ.2).OR.(K2F.EQ.4.AND.K5F.EQ.2)) GO TO 505
      IF((K2C.EQ.3.AND.K5C.EQ.2).OR.(K2F.EQ.3.AND.K5F.EQ.2)) UM1=U(IP,J)
      GO TO 506
  505 UM1=-U(IP,J)
  506 IF (JJJJ.EQ.2) GO TO 510
      K2A=CS(IM,J).AND.MSK(2)
      K2A=K2A*DV(2)
      K2B=CS(IP,J).AND.MSK(2)
      K2B=K2B*DV(2)
      K2E=CS(IM,JP).AND.MSK(2)
      K2E=K2E*DV(2)
      K2G=CS(IP,JP).AND.MSK(2)
      K2G=K2G*DV(2)
      K5A=CS(IM,J).AND.MSK(2)
      K5A=K5A*DV(5)
      K5B=CS(IP,J).AND.MSK(5)
      K5B=K5B*DV(5)
      K5E=CS(IM,JP).AND.MSK(5)
      K5E=K5E*DV(5)
      K5G=CS(IP,JP).AND.MSK(5)
      K5G=K5G*DV(5)
      IF((K2B.EQ.4.AND.K5B.EQ.2).OR.(K2G.EQ.4.AND.K5G.EQ.2)) GO TO 507
      IF((K2B.EQ.3.AND.K5B.EQ.2).OR.(K2G.EQ.3.AND.K5G.EQ.2)) VP1=V(I,JP)
      GO TO 508
  507 VP1=-V(I,JP)
  508 IF((K2A.EQ.4.AND.K5A.EQ.2).OR.(K2F.EQ.4.AND.K5E.EQ.2)) GO TO 509
      IF((K2A.EQ.3.AND.K5A.EQ.2).OR.(K2F.EQ.3.AND.K5E.EQ.2)) VM1=V(I,JP)
      GO TO 510
  509 VM1=-V(I,JP)
  510 IF (IIII.EQ.2) GO TO 515
      VAVE1=.5*(V(I,J)+V(IP,J))
      VAVE2=.5*(V(I,JP)+V(IP,JP))
      AR1=.5*(R(I,J)+R(IP,J))
      IF (VAVE1.LE.0) GO TO 511
      K1C=CS(I,JM).AND.MSK
      K1G=CS(IP,JM).AND.MSK
      IF (K1C.EQ.3.OR.K1G.EQ.3) GO TO 511
      RV1=.5*(R(I,JM)+R(IP,JM))*UM1*VAVE1
      GO TO 512
  511 RV1=AR1*U(IP,J)*VAVE1
  512 IF (VAVE2.GE.0) GO TO 513
      K1D=CS(I,JP).AND.MSK
      K1H=CS(IP,JP).AND.MSK
      IF (K1D.EQ.3.OR.K1H.EQ.3) GO TO 513
      RV2=.5*(R(I,JP)+R(IP,JP))*UP1*VAVE2
      GO TO 514
  513 RV2=AR1*U(IP,J)*VAVE2
  514 U1=U(I,J)+U(IP,J)
      U2=U(IP,J)+U(I+2,J)
      PSI(IP,J)=AR1*U(IP,J)+DT4DX*(R(I,J)*U1*U1-R(IP,J)*U2*U2)+DTDY*(RV1
     $          -RV2)+DTDXS*(MU(IP,J)*(U(I+2,J)-U(IP,J))-MU(I,J)*(U(IP,J
     $          )-U(I,J)))+DT4DY*((MU(I,J)+MU(I,JP)+MU(IP,JP)+MU(IP,J))*
     $          (ODY*(UP1-U(IP,J))+ODX*(V(IP,JP)-V(I,JP)))-(MU(I,JM)+MU(
     $          I,J)+MU(IP,J)+MU(IP,JM))*(ODY*(U(IP,J)-UM1)+ODX*(V(IP,J)
     $          -V(I,J))))+AR1*GXDT
  515 IF (JJJJ.EQ.2) GO TO 520
      UAVE1=.5*(U(I,J)+U(I,JP))
      UAVE2=.5*(U(IP,J)+U(IP,JP))
      AR2=.5*(R(I,J)+R(I,JP))
      IF (UAVE1.LE.0) GO TO 516
      K1A=CS(IM,J).AND.MSK
      K1E=CS(IM,JP).AND.MSK
      IF (K1A.EQ.3.OR.K1E.EQ.3) GO TO 516
      RU1=.5*(R(IM,J)+R(IM,JP))*VM1*UAVE1
      GO TO 517
  516 RU1=AR2*V(I,JP)*UAVE1
  517 IF (UAVE2.GE.0) GO TO 518
      K1B=CS(IP,J).AND.MSK
      K1H=CS(IP,JP).AND.MSK
      IF (K1B.EQ.3.OR.K1H.EQ.3) GO TO 518
      RU2=.5*(R(IP,J)+R(IP,JP))*VP1*UAVE2
      GO TO 519
  518 RU2=AR2*V(I,JP)*UAVE2
  519 V1=V(I,J)+V(I,JP)
      V2=V(I,JP)+V(I,J+2)
      ZET(I,JP)=AR2*V(I,JP)+DTDX*(RU1-RU2)+DT4DY*(R(I,J)*V1*V1-R(I,JP)*V
     $          2*V2)+DTDYS*(MU(I,JP)*(V(I,J+2)-V(I,JP))-MU(I,J)*(V(I,JP
     $          )-V(I,J)))+DT4DX*((MU(I,J)+MU(I,JP)+MU(IP,JP)+MU(IP,J))*
     $          (ODY*(U(IP,JP)-U(IP,J))+ODX*(VP1-V(I,JP)))-(MU(IM,J)+MU(
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     $          IM,JP)+MU(I,JP)+MU(I,J))*(CDY*(U(I,JP)-U(I,J))+CDX*(V(I,
     $          JP)-VM[illegible]))+AN2*GY)
 520 CONTINUE
     DO 524 J=2,NJM1
     JM=J-1
     JP=J+1
     DO 524 I=2,NIM1
     IM=I-1
     IP=I+1
     K4=CS(I,J).AND.MSK(4)
     K4=K4*DV(4)
     IF (K4.EQ.1) GO TO 524
     K2B=CS(IP,J).AND.MSK(2)
     K2B=K2B*DV(2)
     IF (K2B.NE.2) GO TO 521
     PSI(IP,J)=PSI(I,J)
     GO TO 524
 521 K2A=CS(IM,J).AND.MSK(2)
     K2A=K2A*DV(2)
     IF (K2A.NE.2) GO TO 522
     PSI(I,J)=PSI(IP,J)
     GO TO 524
 522 K2D=CS(I,JP).AND.MSK(2)
     K2D=K2D*DV(2)
     IF (K2D.NE.2) GO TO 523
     ZET(I,JP)=ZET(I,J)
     GO TO 524
 523 K2C=CS(I,JM).AND.MSK(2)
     K2C=K2C*DV(2)
     IF (K2C.EQ.2) ZET(I,J)=ZET(I,JP)
 524 CONTINUE
 525 DO 531 J=2,NJM1
     JM=J-1
     JP=J+1
     DO 531 I=2,NIM1
     IM=I-1
     IP=I+1
     K1=CS(I,J).AND.MSK
     IF (K1.NE.4) GO TO 531
     IF (KKKK.EQ.1) GO TO 526
     K12=CS(I,J).AND.MSK(12)
     K12=K12*DV(12)
     K12A=CS(IM,J).AND.MSK(12)
     K12A=K12A*DV(12)
     K12B=CS(IP,J).AND.MSK(12)
     K12B=K12B*DV(12)
     K12C=CS(I,JM).AND.MSK(12)
     K12C=K12C*DV(12)
     K12D=CS(I,JP).AND.MSK(12)
     K12D=K12D*DV(12)
     IF (K12.EQ.1.AND.K12A.EQ.1.AND.K12B.EQ.1.AND.K12C.EQ.1.AND.K12D.EQ
    $ .1) GO TO 531
 526 AR1=1./(R(I,J)+R(IP,J))
     AR2=1./(R(I,J)+R(IM,J))
     AR3=1./(R(I,J)+R(I,JP))
     AR4=1./(R(I,J)+R(I,JM))
     CIJ=1./(DT2*(CDXS*(AR1+AR2)+CDYS*(AR3+AR4)))
     B1(I,J)=CIJ*DTDXS*AR1
     B2(I,J)=CIJ*DTDXS*AR2
     B3(I,J)=CIJ*DTDYS*AR3
     B4(I,J)=CIJ*DTDYS*AR4
     S1=PSI(IP,J)
     S2=PSI(I,J)
     S3=ZET(I,JP)
     S4=ZET(I,J)
     K5B=CS(IP,J).AND.MSK(5)
     K5B=K5B*DV(5)
     IF (K5B.EQ.1) GO TO 527
     K4C=CS(IP,JP).AND.MSK(4)
     K4C=K4C*DV(4)
     IF (K4C.EQ.2) GO TO 300
     S4=S4-DT4DY*(MU(I,J)+MU(I,J)+MU(I,JM)+MU(IP,JM))*U(IP,JM)*CDX
     GO TO 301
 300 S3=S3-DT4DY*(MU(I,J)+MU(I,J)+MU(I,JP)+MU(IP,JP))*U(IP,JP)*CDX
 301 S1=R(I,J)*GXDT
     K2B=CS(IP,J).AND.MSK(2)
     K2B=K2B*DV(2)
     IF (K2B.EQ.4) S1=S1+DXI4*MU(I,J)*U(I,J)+CDY*[illegible]JP)*(MU(I,J)+
    $          MU(I,JP))-V(I,J)*(MU(I,J)+MU[illegible]M)))
 527 K5A=CS(IM,J).AND.MSK(5)
     K5A=K5A*DV(5)
     IF (K5A.EQ.1) GO TO 528
     K4A=CS(IM,JP).AND.MSK(4)
     K4A=K4A*DV(4)
     IF (K4A.EQ.2) GO TO 302
     S4=S4+DT4DY*(MU(I,J)+MU(I,J)+MU(IM,JM)+MU(I,JM))*U(I,JM)*CDY
     GO TO 303
 302 S3=S3+DT4DY*(MU(I,J)+MU(I,J)+MU(IM,JP)+MU(I,JP))*U(I,JP)*CDY
 303 S2=R(I,J)*GXDT
     K2A=CS(IM,J).AND.MSK(2)
     K2A=K2A*DV(2)
     IF (K2A.EQ.4) S2=S2+DXI4*MU(I,J)*U(IP,J)+CDY*[illegible],JP)*(MU(I,J)+
    $          MU(I,JP))-V(I,J)*(MU(I,J)+MU(I,JM)))
 528 K5D=CS(I,JP).AND.MSK(5)
     K5D=K5D*DV(5)
     IF (K5D.EQ.1) GO TO 529
     K4C=CS(IP,JP).AND.MSK(4)
     K4C=K4C*DV(4)
     IF (K4C.EQ.2) GO TO 304
     S2=S2-DT4DX*(MU(I,[illegible])+MU(IM,JP))*V(IM,JP)*CDY
     GO TO 305
 304 S1=S1-DT4DX*(MU(I,J)+MU(I,J)+MU(IP,J)+MU(IP,JP))*V(IP,JP)*CDY
 305 S3=R(I,J)*GYDT
     K2D=CS(I,JP).AND.MSK(2)
     K2D=K2D*DV(2)
     IF (K2D.EQ.4) S4=S4+DYI4*MU(I,J)*V(I,J)+CDX*(U(IP,J)*(MU(I,J)+
    $          MU(IP,J))-U(I,J)*(MU(I,J)+[illegible])))
 529 K5C=CS(I,JM).AND.MSK(5)
     K5C=K5C*DV(5)
     IF (K5C.EQ.1) GO TO 530
     K4B=CS(IP,JM).AND.MSK(4)
     K4B=K4B*DV(4)
     IF (K4B.EQ.2) GO TO 306
     S2=S2+DT4DX*(MU(I,J)+MU(I,J)+MU(IM,JM)+MU(IM,[illegible](IM,J)*CDY
     GO TO 307
 306 S1=S1+DT4DX*(MU(I,J)+MU(I,J)+MU(IP,JM)+MU(IP,J))*V(IP,J)*CDY
 307 S4=R(I,J)*GYDT
     K2C=CS(I,JM).AND.MSK(2)
     K2C=K2C*DV(2)
     IF (K2C.EQ.4) S4=S4+DYI4*MU(I,J)*V(I,JP)+CDX*(U(IP,J)*(MU(I,J)+
    $          MU(IP,J))-U(I,J)*(MU(I,J)+MU[illegible]))
 530 A(I,J)=CIJ*(CDX2*(S2*AR2-S1*AR1)+CDY2*(S4*AR4-S3*AR3))
 531 CONTINUE
     RETURN
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      END
      SUBROUTINE PRSITN
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,TWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNAP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVF,XI,XA,IWNG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DIMENSION PN(40,20)
      EQUIVALENCE (PN,SRT)
      DO 200 J=1,NJ
      DO 200 I=1,NI
  200 PN(I,J)=P(I,J)
      ICNTP=0
      Q=1.
  201 ERR=0.
      DO 213 L=1,ITEST
      ICNTP=ICNTP+1
      RESID=0.
      DO 211 J=2,NJM1
      JM=J-1
      JP=J+1
      DO 211 I=2,NIM1
      IM=I-1
      IP=I+1
      K1=CS(I,J).AND.MSK
      IF (K1.NE.4) GO TO 211
      K4=CS(I,J).AND.MSK(4)
      K4=K4*DV(4)
      IF (K4.EQ.1) GO TO 210
      K1D=CS(I,JP).AND.MSK
      IF (K1D.NE.2) GO TO 204
      K2D=CS(I,JP).AND.MSK(2)
      K2D=K2D*DV(2)
      IF (K2D.EQ.1) GO TO 203
      IF (K2D.EQ.2) GO TO 203
      PN(I, JP)=PN(I,J)+R(I,J)*GYD
      IF (K2D.EQ.3) GO TO 204
      AMIJ=MU(I,J)
      PN(I,JP)=PN(I,JP)+DYI4*AMIJ*VT(I,J)+ODX*(UT(IP,J)*(AMIJ+MU(IP,J))
     $           -UT(I,J)*(AMIJ+MU(IM,J)))
      GO TO 204
  203 PN(I, JP)=PN(I,J)
  204 K1C=CS(I,JM).AND.MSK
      IF (K1C.NE.2) GO TO 207
      K2C=CS(I,JM).AND.MSK(2)
      K2C=K2C*DV(2)
      IF (K2C.EQ.1) GO TO 206
      IF (K2C.EQ.2) GO TO 206
      PN(I, JM)=PN(I,J)-R(I,J)*GYD
      IF (K2C.EQ.3) GO TO 207
      AMIJ=MU(I,J)
      PN(I,JM)=PN(I,JM)-DYI4*AMIJ*VT(I,JP)-ODX*(UT(IP,J)*(AMIJ+MU(IP,J))
     $           -UT(I,J)*(AMIJ+MU(IM,J)))
      GO TO 207
  206 PN(I, JM)=PN(I,J)
  207 K1B=CS(IP,J).AND.MSK
      IF (K1B.NE.2) GO TO 1
      K2B=CS(IP,J).AND.MSK(2)
      K2B=K2B*DV(2)
      IF (K2B.EQ.1) GO TO 209
      IF (K2B.EQ.2) GO TO 209
      PN(IP,J)=PN(I,J)+R(I,J)*GXD
      IF (K2B.EQ.3) GO TO 1
      AMIJ=MU(I,J)
      PN(IP,J)=PN(IP,J)+DXI4*AMIJ*UT(I,J)+ODY*(VT(I,JP)*(AMIJ+MU(I,JP))
     $           -VT(I,J)*(AMIJ+MU(I,JM)))
      GO TO 1
  209 PN(IP,J)=PN(I,J)
    1 K1A=CS(IM,J).AND.MSK
      IF (K1A.NE.2) GO TO 210
      K2A=CS(IM,J).AND.MSK(2)
      K2A=K2A*DV(2)
      IF (K2A.EQ.1) GO TO 3
      IF (K2A.EQ.2) GO TO 3
      PN(IM,J)=PN(I,J)-R(I,J)*GXD
      IF (K2A.EQ.3) GO TO 210
      AMIJ=MU(I,J)
      PN(IM,J)=PN(IM,J)-DXI4*AMIJ*UT(IP,J)-ODY*(VT(I,JP)*(AMIJ+MU(I,JP))
     $           -VT(I,J)*(AMIJ+MU(I,JM)))
      GO TO 210
    3 PN(IM,J)=PN(I,J)
  210 RESIDN =B1(I,J)*PN(IP,J)+B2(I,J)*PN(IM,J)+B3(I,J)*PN(I,JP)+B4(I,J)
     $          *PN(I,JM)+A(I,J)-PN(I,J)
      PN(I,J)=PN(I,J)+Q*RESIDN
      RESID=AMAX1(ABS(RESIDN),RESID)
      IF (L.EQ.ITEST) ERR=AMAX1(ERR,ABS(PN(I,J)-P(I,J))*GH/ABS(R(I,J)))
  211 CONTINUE
      DO 212 J=1,NJ
      DO 212 I=1,NI
  212 P(I,J)=PN(I,J)
      IF (RESID.EQ.0) GO TO 213
      IF (ICNTP.EQ.11) RESIDL=RESID
      IF (ICNTP.EQ.12) Q=2./(1.+SQRT(1.-RESID/RESIDL))
  213 CONTINUE
      IF (ERR.LT.EPS) GO TO 214
      IF (ICNTP.LT.100*ITEST) GO TO 201
      KD=2
      WRITE (MO,2000) T,ICNTR,LL
      GO TO 215
  214 WRITE (MO,2001) ICNTP,T,ICNTR,LL
  215 RETURN
 2000 FORMAT(1H-/1H-,20X30HTOO MANY ITERATIONS AT TIME = ,F4.3,10X8HICNT
     $R = ,I4,10X5HLL = ,I4)
 2001 FORMAT(1H ,10X,I4,23H  ITERATIONS AT TIME = ,F6.3,10X8HICNTR = ,I4
     $,10X5HLL = ,I4)
      END
      SUBROUTINE DENCHG
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IPT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXT4,DXIN,DXP,DYC,DYCD2,DYT4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
     $       V0,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNAP,NAP,NAP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      LL=0
      DO 600 J=1,NJ
      DO 600 I=1,NI
  600 CS(I,J)=IMP(12).OR.(CS(I,J).AND.MKC(12))
      DO 602 J=2,NJM1
      DO 602 I=2,NIM1
      K1=CS(I,J).AND.MSK
      IF (K1.EQ.4) GO TO 601
      IF (K1.NE.5) GO TO 602
  601 K11=CS(I,J).AND.MSK(11)
      K11=K11*DV(11)
      IF (K11.EQ.1) GO TO 602
      IF (NK(I,J)+NKT(I,J).EQ.0.) GO TO 602
      RHO=(SR(I,J)+SRT(I,J))/(NK(I,J)+NKT(I,J))
  699 IF (RHO.EQ.R(I,J)) GO TO 602
      R(I,J)=RHO
      LL=LL+1
      KT=2*IMP(12)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(12))
      KT=2*IMP(13)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(13))
  602 CONTINUE
      RETURN
      END
      SUBROUTINE MOVPAR
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXT4,DXIN,DXP,DYC,DYCD2,DYT4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
     $       V0,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNAP,NAP,NAP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DO 148 K=1,NP
      KP=PS(K).AND.MSK
      IF (KP.EQ.3) GO TO 148
      MM=1
   98 O=XP(K)*ODX+2.
      Q=YP(K)*ODY+2.
      I=O
      J=Q
      FX=O-I
      FY=Q-J
      K1=CS(I,J).AND.MSK
      IF (K1.EQ.4) GO TO 99
      IF (K1.NE.5) GO TO 138
   99 IF (KKK.EQ.2) GO TO 100
      K10=CS(I,J).AND.MSK(10)
      K10=K10*DV(10)
      IF (K10.EQ.1) GO TO 148
  100 IF (FY.LT..5) GO TO 101
      JPR=J
      GO TO 102
  101 JPR=J-1
  102 IP=I+1
      JPRP=JPR+1
      HPSX=.5+ODX*(XC(I)-XP(K))
      HPSY=.5+ODY*(YC(JPR)+DYCD2-YP(K))
      HMSX=1.-HPSX
      HMSY=1.-HPSY
      UT1=UT(I,JPRP)
      UT2=UT(I,J)
      UT3=UT(IP,JPRP)
      UT4=UT(IP,J)
      UT5=UT(I,JPR)
      UT6=UT(IP,JPR)
  104 IF (UT1.NE.0.) GO TO 105
      U1=UT2
      GO TO 107
  105 K5A=CS(I-1,J).AND.MSK(5)
      K5A=K5A*DV(5)
      IF (K5A.EQ.1) GO TO 106
      U1=0.
      GO TO 107
  106 U1=UT1
  107 IF (UT3.NE.0.) GO TO 108
      U2=UT4
      GO TO 110
  108 K5B=CS(IP,J).AND.MSK(5)
      K5B=K5B*DV(5)
      IF (K5B.EQ.1) GO TO 109
      U2=0.
      GO TO 110
  109 U2=UT3
  110 IF (UT5.NE.0.) GO TO 111
      U3=UT2
      GO TO 113
  111 K5A=CS(I-1,J).AND.MSK(5)
      K5A=K5A*DV(5)
      IF (K5A.EQ.1) GO TO 112
      U3=0.
      GO TO 113
  112 U3=UT5
  113 IF (UT6.NE.0.) GO TO 114
```

```
    U4=UT4
    GO TO 116
114 K5B=CS(IP,J).AND.MSK(5)
    K5B=K5B*DV(5)
    IF (K5B.EQ.1) GO TO 115
    U4=0.
    GO TO 116
115 U4=UT4
116 UPT=HPSX*HMSY*U1+HMSX*HMSY*U2+HPSX*HPSY*U3+HMSX*HPSY*U4
    IF (MM.EQ.2) XP(K)=XP(K)-S*DXP
    XPT=XP(K)+UPT*DT
    IF (FX.LT..5) GO TO 117
    IPR=I
    GO TO 118
117 IPR=I-1
118 JP=J+1
    IPRP=IPR+1
    HPSX=.5+CDX*(XC(IPR)+DXCD2-XP(K))
    HPSY=.5+CDY*(YC(J)-YP(K))
    HMSX=1.-HPSX
    HMSY=1.-HPSY
    VT1=VT(IPR,JP)
    VT2=VT(I,JP)
    VT3=VT(IPRP,JP)
    VT4=VT(IPR,J)
    VT5=VT(I,J)
    VT6=VT(IPRP,J)
120 IF (VT1.NE.0.) GO TO 121
    V1=VT2
    GO TO 123
121 K5C=CS(I,JP).AND.MSK(5)
    K5C=K5C*DV(5)
    IF (K5C.EQ.1) GO TO 122
    V1=0.
    GO TO 123
122 V1=VT1
123 IF (VT3.NE.0.) GO TO 124
    V2=VT2
    GO TO 126
124 K5C=CS(I,JP).AND.MSK(5)
    K5C=K5C*DV(5)
    IF (K5C.EQ.1) GO TO 125
    V2=0.
    GO TO 126
125 V2=VT3
126 IF (VT4.NE.0.) GO TO 127
    V3=VT5
    GO TO 129
127 K5D=CS(I,J-1).AND.MSK(5)
    K5D=K5D*DV(5)
    IF (K5D.EQ.1) GO TO 128
    V3=0.
    GO TO 129
128 V3=VT4
129 IF (VT6.NE.0.) GO TO 130
    V4=VT5
    GO TO 132
130 K5D=CS(I,J-1).AND.MSK(5)
    K5D=K5D*DV(5)
    IF (K5D.EQ.1) GO TO 131
    V4=0.
    GO TO 132
131 V4=VT6
132 VPT=HPSX*HMSY*V1+HMSX*HMSY*V2+HPSX*HPSY*V3+HMSX*HPSY*V4
    IF (MM.EQ.3) YP(K)=YP(K)-S*DYP
    YPT=YP(K)+VPT*DT
    I1=XPT*CDX+2.
    J1=YPT*CDY+2.
    IF(MM.EQ.2)I=I-S
    IF(MM.EQ.3)J=J-S
    IF (I1.LE.NI.AND.J1.LE.NJ.AND.I1.GE.1.AND.J1.GE.1) GO TO 300
    GO TO 301
300 K1=CS(I,J).AND.MSK
    IF (K1.EQ.1) GO TO 301
    IF (K1.NE.2) GO TO 133
    K2=CS(I,J).AND.MSK(2)
    K2=K2*DV(2)
    KP=PS(K).AND.MSK
    IF (K2.EQ.1.AND.KP.EQ.2) GO TO 133
301 PS(K)=3.OR.(PS(K).AND.MKC(3))
    KT=IMP(5)
    PS(K)=KT.OR.(PS(K).AND.MKC(5))
    XP(K)=0.
    YP(K)=0.
    UP(K)=0.
    VP(K)=0.
    GO TO 148
133 IF (KKK.EQ.1) GO TO 134
    UP(K)=UPT
    XP(K)=XPT
    VP(K)=VPT
    YP(K)=YPT
    GO TO 148
134 IF (J.NE.J1) GO TO 135
    IF (I.EQ.I1) GO TO 148
135 KPA=PS(K).AND.MSK(3)
    KPA=KPA*DV(3)
    IF (KPA.EQ.2) GO TO 136
    SRT(I,J)=SRT(I,J)-R1
    SRT(I1,J1)=SRT(I1,J1)+R1
    GO TO 137
136 SRT(I,J)=SRT(I,J)-R2
    SRT(I1,J1)=SRT(I1,J1)+R2
137 NKT(I,J)=NKT(I,J)-1.
    NKT(I1,J1)=NKT(I1,J1)+1.
    GO TO 148
138 K2=CS(I,J).AND.MSK(2)
    K2=K2*DV(2)
    IF (K2.NE.1) GO TO 148
    IF (IVC.EQ.0) GO TO 4
    IF (KKK.EQ.2) GO TO 200
    K10=CS(I,J).AND.MSK(10)
    K10=K10*DV(10)
    IF (K10.EQ.1) GO TO 148
200 K7=CS(I,J).AND.MSK(7)
    K7=K7*DV(7)
    IF (K7.EQ.4) GO TO 3
    IF (K7.EQ.3) GO TO 2
    IF (K7.EQ.2) GO TO 1
    K1A=CS(I+1,J).AND.MSK
    IF (K1A.EQ.3) GO TO 148
    XP(K)=XP(K)+DXP
    MM=2
    S=1.
```

```
      GO TO 98
    1 K1B=CS(I-1,J).AND.MSK
      IF (K1B.EQ.3) GO TO 148
      XP(K)=XP(K)-DXP
      MM=2
      S=-1.
      GO TO 98
    2 K1C=CS(I,J+1).AND.MSK
      IF (K1C.EQ.3) GO TO 148
      YP(K)=YP(K)+DYP
      MM=3
      S=1.
      GO TO 98
    3 K1D=CS(I,J-1).AND.MSK
      IF (K1D.EQ.3) GO TO 148
      YP(K)=YP(K)-DYP
      MM=3
      S=-1.
      GO TO 98
    4 UT7=UT(I,J)
      UT8=UT(I+1,J)
      VT7=VT(I,J)
      VT8=VT(I,J+1)
  140 K7=CS(I,J).AND.MSK(7)
      K7=K7/DV(7)
      IF (K7.EQ.1) GO TO 144
      IF (K7.EQ.2) GO TO 143
      IF (K7.EQ.3) GO TO 141
      T2=VT7*DT
      DYIN=-T2
      KT=4*IMP(2)
      PS(K)=KT.OR.(PS(K).AND.MKC(2))
      GO TO 142
  141 T2=VT8*DT
      DYIN=T2
      KT=3*IMP(2)
      PS(K)=KT.OR.(PS(K).AND.MKC(2))
  142 YPT=YP(K)+T2
      XPT=XP(K)
      GO TO 146
  143 T1=UT7*DT
      DXIN=-T1
      KT=2*IMP(2)
      PS(K)=KT.OR.(PS(K).AND.MKC(2))
      GO TO 145
  144 T1=UT8*DT
      DXIN=T1
      PS(K)=IMP(2).OR.(PS(K).AND.MKC(2))
  145 XPT=XP(K)+T1
      YPT=YP(K)
  146 IF (KKK.EQ.2) GO TO 147
      I1=XPT*ODX+2.
      J1=YPT*ODY+2.
      GO TO 134
  147 XP(K)=XPT
      YP(K)=YPT
  148 CONTINUE
  149 RETURN
      END
      SUBROUTINE VELCTS(KKKK)
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MC,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
     $       V0,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DO 406 J=2,NJM1
      JP=J+1
      DO 406 I=2,NIM1
      IP=I+1
      K1=CS(I,J).AND.MSK
      IF (K1.EQ.4) GO TO 400
      IF (K1.NE.5) GO TO 406
  400 K1B=CS(IP,J).AND.MSK
      IF (K1B.NE.4.AND.K1B.NE.5) GO TO 403
  402 T1=(PSI(IP,J)+DTDX*(P(I,J)-P(IP,J)))/(R(I,J)+R(IP,J))
      UT(IP,J)=T1+T1
  403 K1D=CS(I,JP).AND.MSK
      IF (K1D.NE.4.AND.K1D.NE.5) GO TO 406
  405 T2=(ZET(I,JP)+DTDY*(P(I,J)-P(I,JP)))/(R(I,J)+R(I,JP))
      VT(I,JP)=T2+T2
  406 CONTINUE
      CALL BNDCND(KKKK)
      RETURN
      END
      SUBROUTINE DENVIS
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,LPEF,ITAL,IWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MC,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,U0,
     $       V0,W,IVO,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),B1(40,20),B2(40,20),B3(40,20),B4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DIMENSION SM(40,20)
      EQUIVALENCE (SM,NKT)
      DO 300 J=1,NJ
      DO 300 I=1,NI
      NK(I,J)=0.
      SR(I,J)=0.
  300 SM(I,J)=0.
      KK=1
      DO 303 K=1,NP
      KP=PS(K).AND.MSK
      IF (KP.EQ.3) GO TO 303
      KK=2
      I=XP(K)*ODX+2.
      J=YP(K)*ODY+2.
      KPA=PS(K).AND.MSK(3)
      KPA=KPA*DV(3)
      IF (KPA.EQ.2) GO TO 301
      SR(I,J)=SR(I,J)+R1
      SM(I,J)=SM(I,J)+MU1
      GO TO 302
  301 SR(I,J)=SR(I,J)+R2
      SM(I,J)=SM(I,J)+MU2
  302 NK(I,J)=NK(I,J)+1.
  303 CONTINUE
      IF (KK.EQ.1) GO TO 306
      DO 305 J=1,NJ
      DO 305 I=1,NI
      IF (NK(I,J).EQ.0.) GO TO 305
      R(I,J)=SR(I,J)/NK(I,J)
      MU(I,J)=SM(I,J)/NK(I,J)
  305 CONTINUE
      DO 357 J=1,NJ
      JM=J-1
      JP=J+1
      DO 357 I=1,NI
      IM=I-1
      IP=I+1
      K1=CS(I,J).AND.MSK
      IF (K1.NE.2) GO TO 353
      K7=CS(I,J).AND.MSK(7)
      K7=K7*DV(7)
      IF (K7.EQ.4) GO TO 352
      IF (K7.EQ.3) GO TO 351
      IF (K7.EQ.2) GO TO 350
      R(I,J)=R(IP,J)
      MU(I,J)=MU(IP,J)
      GO TO 357
  350 R(I,J)=R(IM,J)
      MU(I,J)=MU(IM,J)
      GO TO 357
  351 R(I,J)=R(I,JP)
      MU(I,J)=MU(I,JP)
      GO TO 357
  352 R(I,J)=R(I,JM)
      MU(I,J)=MU(I,JM)
      GO TO 357
  353 IF (K1.NE.1) GO TO 357
      K7=CS(I,J).AND.MSK(7)
      K7=K7*DV(7)
      IF (K7.EQ.4) GO TO 356
      IF (K7.EQ.3) GO TO 355
      IF (K7.EQ.2) GO TO 354
      IF (K7.NE.1) GO TO 357
      R(I,J)=R(IP,JP)
      MU(I,J)=MU(IP,JP)
      GO TO 357
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  354 R(I,J)=R(IM,JM)
      MU(I,J)=MU(IM,JM)
      GO TO 357
  355 R(I,J)=R(IM,JP)
      MU(I,J)=MU(IM,JP)
      GO TO 357
  356 R(I,J)=R(IP,JM)
      MU(I,J)=MU(IP,JM)
  357 CONTINUE
      GO TO 307
  306 KD=2
      WRITE (MO,3000) T
  307 RETURN
 3000 FORMAT(1H-/1H-,20X33HNO PARTICLES IN SYSTEM AT TIME = ,F6.3)
      END
      SUBROUTINE REFCEL
      COMMON/TVPOOL/PLMIN,PLMAX,XMIN,XMAX,TXMIN,TXMAX,TYMIN,TYMAX
      COMMON/TVGUIDE/TMODE,TEXT,ITV
      COMMON/TVFACT/FACT
      COMMON/TVTUNE/LPEN,IPEF,ITAL,TWINK,INTS,IRT,IUP
      COMMON DELTAS,DT,DT2,DT4DX,DT4DY,DTCP,DTDX,DTDY,DTDXS,DTDYS,DTP,
     $       DTPP,DTVP,DXC,DXCD2,DXI4,DXIN,DXP,DYC,DYCD2,DYI4,DYIN,DYP
      COMMON EPS,G,GH,GX,GXD,GXDT,GY,GYD,GYDT,H,ICNTR,ITEST,KD,KKK,LL,
     $       MI,MO,MU1,MU2,NB1,NB2,NI,NIM1,NIP1,NJ,NJM1,NJP1,NP,NPR,NTP,
     $       ODX,ODX2,ODXS,ODY,ODY2,ODYS,R1,R2,T,TCP,TL,TP,TPP,TVP,UO,
     $       VO,W,IVC,DXO2DY,DYO2DX,DXODY,DYODX
      COMMON A(40,20),A1(40,20),A2(40,20),A3(40,20),A4(40,20),CS(40,20),
     $       DV(15),IMP(15),MKC(15),MSK(15),MU(40,20),NF(30),NK(40,20),
     $       NKT(40,20),P(40,20),PS(3000),PSI(41,21),R(40,20),SR(40,20),
     $       SRT(40,20),U(41,21),UP(3000),UT(41,21),V(41,21),VP(3000),
     $       VT(41,21),XC(40),XP(3000),XPO(30),XPL(30),YC(20),YP(3000),
     $       YPO(30),YPL(30),ZET(41,21),UPO(30),VPO(30)
      COMMON KNBP,NBP,NBP2,XB1(21),XB2(21),YB1(21),YB2(21),XX1(20),
     $       XX2(20),ISAVE,XI,XA,IWDG,DTPR,TPR,NC,ISPACE,DUM(5)
      REAL MU,MU1,MU2,NK,NKT
      INTEGER CS,PS
      DIMENSION SU(41,21),SV(41,21)
      IF (T.EQ.0) GO TO 1
      DO 710 K=1,NP
      KP=PS(K).AND.MSK
      IF (KP.EQ.3) GO TO 710
      IF (KP.NE.2) GO TO 710
      I=XP(K)*ODX+2.
      J=YP(K)*ODY+2.
  701 K2=CS(I,J).AND.MSK(2)
      K2=K2*DV(2)
      IF (K2.EQ.1) GO TO 710
      DO 702 L=1,NP
      LP=PS(L).AND.MSK
      IF (LP.EQ.3) GO TO 703
  702 CONTINUE
      GO TO 710
  703 KB=PS(K).AND.MSK(2)
      KB=KB*DV(2)
      KPA=PS(K).AND.MSK(3)
      KPA=KPA*DV(3)
      IF(KPA.EQ.1) GO TO 698
      IF(ISPACE.LE.0) GO TO 698
      DXIN=DXP*.5
      DYIN=DYP*.5
      GO TO 699
  698 DXIN=DXP
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      DYIN=DYP
  699 IF (K9.NE.4) GO TO 704
      IF (VP(K).GE.0) GO TO 710
      YP(L)=YP(K)+DYIN
      GO TO 705
  704 IF (K9.NE.3) GO TO 706
      IF (VP(K).LE.0) GO TO 710
      YP(L)=YP(K)-DYIN
  705 XP(L)=XP(K)
      GO TO 709
  706 IF (K8.NE.1) GO TO 707
      IF (UP(K).LE.0) GO TO 710
      XP(L)=XP(K)-DXIN
      GO TO 708
  707 IF (UP(K).GE.0) GO TO 710
      XP(L)=XP(K)+DXIN
  708 YP(L)=YP(K)
  709 PS(L)=2.OR.(PS(L).AND.MKC)
      PS(K)=1.OR.(PS(K).AND.MKC)
      KT=K8*IMP(2)
      PS(L)=KT.OR.(PS(L).AND.MKC(2))
      K3=PS(K).AND.MSK(3)
      K3=K3*DV(3)
      KT=K3*IMP(3)
      PS(L)=KT.OR.(PS(L).AND.MKC(3))
      K4=PS(L).AND.MSK(4)
      K4=K4*DV(4)+1
      IF (K4.EQ.8) K4=1
      KT=K4*IMP(4)
      PS(L)=KT.OR.(PS(L).AND.MKC(4))
  710 CONTINUE
      DO 711 J=1,NJ
      DO 711 I=1,NI
      KT=4*IMP(9)
  711 CS(I,J)=KT.OR.(CS(I,J).AND.MKC(9))
      DO 712 J=1,NJ
      DO 712 I=1,NI
      SU(I,J)=0.
  712 SV(I,J)=0.
      DO 714 K=1,NP
      KP=PS(K).AND.MSK
      IF (KP.EQ.3) GO TO 714
      I=XP(K)*ODX+2.
      J=YP(K)*ODY+2.
      IF (I.LT.1.OR.J.LT.1.OR.I.GT.NI.OR.J.GT.NJ) GO TO 2
      K1=CS(I,J).AND.MSK
      IF (K1.EQ.1) GO TO 2
      IF (K1.NE.2) GO TO 3
      K2=CS(I,J).AND.MSK(2)
      K2=K2*DV(2)
      IF (K2.NE.1) GO TO 2
    3 SU(I,J)=SU(I,J)+UP(K)
      SV(I,J)=SV(I,J)+VP(K)
      K9=CS(I,J).AND.MSK(9)
      K9=K9*DV(9)
      KPA=PS(K).AND.MSK(3)
      KPA=KPA*DV(3)
      IF (K9.NE.4) GO TO 713
      KT=KPA*IMP(9)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(9))
      GO TO 714
  713 IF (KPA.EQ.K9) GO TO 714
      KT=3*IMP(9)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(9))
      GO TO 714
    2 PS(K)=3.OR.(PS(K).AND.MKC)
      KT=IMP(5)
      PS(K)=KT.OR.(PS(K).AND.MKC(5))
      XP(K)=0.
      YP(K)=0.
      UP(K)=0.
      VP(K)=0.
  714 CONTINUE
      DO 716 J=2,NJM1
      JM=J-1
      JP=J+1
      DO 716 I=2,NIM1
      IM=I-1
      IP=I+1
      K1=CS(I,J).AND.MSK
      IF (K1.NE.5) GO TO 715
      IF (NK(I,J).NE.0) GO TO 716
      CS(I,J)=3.OR.(CS(I,J).AND.MKC)
      KT=4*IMP(9)
      CS(I,J)=KT.OR.(CS(I,J).AND.MKC(9))
      P(I,J)=0.
      R(I,J)=0.
      MU(I,J)=0.
      GO TO 716
  715 IF (K1.NE.3) GO TO 716
      P(I,J)=0.
      K1A=CS(IM,J).AND.MSK
      K1B=CS(IP,J).AND.MSK
      K1C=CS(I,JM).AND.MSK
      K1D=CS(I,JP).AND.MSK
      IF (K1A.EQ.3) UT(I,J)=0.
      IF (K1B.EQ.3) UT(IP,J)=0.
      IF (K1C.EQ.3) VT(I,J)=0.
      IF (K1D.EQ.3) VT(I,JP)=0.
      IF (NK(I,J).EQ.0) GO TO 716
      CS(I,J)=5.OR.(CS(I,J).AND.MKC)
      K1A=CS(IM,J).AND.MSK
      K1B=CS(IP,J).AND.MSK
      K1C=CS(I,JM).AND.MSK
      K1D=CS(I,JP).AND.MSK
      K2A=CS(IM,J).AND.MSK(2)
      K2A=K2A*DV(2)
      K2B=CS(IP,J).AND.MSK(2)
      K2B=K2B*DV(2)
      K2C=CS(I,JM).AND.MSK(2)
      K2C=K2C*DV(2)
      K2D=CS(I,JP).AND.MSK(2)
      K2D=K2D*DV(2)
      IF (K1A.EQ.3.OR.K2A.EQ.2) UT(I,J)=SU(I,J)/NK(I,J)
      IF (K1B.EQ.3.OR.K2B.EQ.2) UT(IP,J)=SU(I,J)/NK(I,J)
      IF (K1C.EQ.3.OR.K2C.EQ.2) VT(I,J)=SV(I,J)/NK(I,J)
      IF (K1D.EQ.3.OR.K2D.EQ.2) VT(I,JP)=SV(I,J)/NK(.,J)
  716 CONTINUE
    1 DO 723 J=2,NJM1
      JM=J-1
      JP=J+1
      DO 723 I=2,NIM1
      IM=I-1
      IP=I+1
```

```fortran
      IF (MK(I,J).EQ.0) GO TO 723
      K1=CS(I,J).AND.MSK
      K1A=CS(IM,J).AND.MSK
      K1B=CS(IP,J).AND.MSK
      K1C=CS(I,JM).AND.MSK
      K1D=CS(I,JP).AND.MSK
      IF (K1.NE.4) GO TO 718
      IF (K1A.EQ.3.OR.K1B.EQ.3.OR.K1C.EQ.3.OR.K1D.EQ.3) GO TO 717
      GO TO 723
  717 CS(I,J)=5.OR.(CS(I,J).AND.MKC)
      P(I,J)=0.
      GO TO 723
  718 P(I,J)=0.
      IF (K1.NE.5) GO TO 723
      SP=0.
      AN=0.
      IF (K1A.EQ.3) GO TO 723
      IF (K1A.NE.4) GO TO 719
      SP=SP+P(IM,J)
      AN=AN+1.
  719 IF (K1B.EQ.3) GO TO 723
      IF (K1B.NE.4) GO TO 720
      SP=SP+P(IP,J)
      AN=AN+1.
  720 IF (K1C.EQ.3) GO TO 723
      IF (K1C.NE.4) GO TO 721
      SP=SP+P(I,JM)
      AN=AN+1.
  721 IF (K1D.EQ.3) GO TO 723
      IF (K1D.NE.4) GO TO 722
      SP=SP+P(I,JP)
      AN=AN+1.
  722 CS(I,J)=4.OR.(CS(I,J).AND.MKC)
      IF (AN.NE.0) P(I,J)=SP/AN
  723 CONTINUE
      CALL BNDCND(1)
      DO 732 J=2,NJM1
      JM=J-1
      JP=J+1
      DO 732 I=2,NIM1
      IM=I-1
      IP=I+1
      K1=CS(I,J).AND.MSK
      IF (K1.NE.5) GO TO 732
      K1A=CS(IM,J).AND.MSK
      K1B=CS(IP,J).AND.MSK
      K1C=CS(I,JM).AND.MSK
      K1D=CS(I,JP).AND.MSK
      IF (K1A.EQ.3.AND.K1B.NE.3.AND.K1C.NE.3.AND.K1D.NE.3) GO TO 726
      IF (K1A.NE.3.AND.K1B.EQ.3.AND.K1C.NE.3.AND.K1D.NE.3) GO TO 726
      IF (K1A.NE.3.AND.K1B.NE.3.AND.K1C.EQ.3.AND.K1D.NE.3) GO TO 727
      IF (K1A.NE.3.AND.K1B.NE.3.AND.K1C.NE.3.AND.K1D.EQ.3) GO TO 727
      IF (K1A.NE.3.AND.K1B.EQ.3.AND.K1C.NE.3.AND.K1D.EQ.3) GO TO 728
      IF (K1A.NE.3.AND.K1B.EQ.3.AND.K1C.EQ.3.AND.K1D.NE.3) GO TO 729
      IF (K1A.EQ.3.AND.K1B.NE.3.AND.K1C.EQ.3.AND.K1D.NE.3) GO TO 730
      IF (K1A.EQ.3.AND.K1B.NE.3.AND.K1C.NE.3.AND.K1D.EQ.3) GO TO 731
      P(I,J)=0.
      GO TO 732
  726 P(I,J)=CDX2*MU(I,J)*(U(IP,J)-U(I,J))
      GO TO 732
  727 P(I,J)=CDY2*MU(I,J)*(V(I,JP)-V(I,J))
      GO TO 732
  728 P(I,J)=MU(I,J)*.5*(CDY*(U(I,J)+U(IP,J)-U(I,JM)-U(IP,JM))+CDX*(
     $        V(I,JP)+V(I,J)-V(IM,JP)-V(IM,J)))
      GO TO 732
  729 P(I,J)=.5*MU(I,J)*(CDY*(U(IP,J)+U(I,J)-U(IP,JP)-U(I,JP))+CDX*(
     $        V(IM,JP)+V(IM,J)-V(I,JP)-V(I,J)))
      GO TO 732
  730 P(I,J)=.5*MU(I,J)*(CDY*(U(IP,JP)+U(I,JP)-U(IP,J)-U(I,J))+CDX*(
     $        V(IP,JP)+V(IP,J)-V(I,JP)-V(I,J)))
      GO TO 732
  731 P(I,J)=.5*MU(I,J)*(CDY*(U(I,JM)+U(IP,JM)-U(I,J)-U(IP,J))+CDX*(
     $        V(I,JP)+V(I,J)-V(IP,JP)-V(IP,J)))
  732 CONTINUE
      CALL FLGCEL
      RETURN
      END
```

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